Complex-Time (Kime)

- At a given spatial location, \( x \), complex time (kime) is defined by \( \kappa = re^{i\theta} \in \mathbb{C} \), where:
  - the magnitude represents the longitudinal events order (\( r > 0 \)) and characterizes the longitudinal displacement in time, and
  - event phase (\( -\pi < \theta < \pi \)) is an angular displacement, or event direction
- There are multiple alternative parametrizations of kime in the complex plane
- Space-time manifold is \( \mathbb{R}^4 \times \mathbb{C} \)
- \((x, \kappa_1)\) and \((x, \kappa_2)\) have the same spactime representation, but different spacetime coordinates,
- \((x, \kappa_1)\) and \((y, \kappa_1)\) share the same kime, but represent different spatial locations,
- \((x, \kappa_2)\) and \((x, \kappa_3)\) have the same spatial locations and kime-directions, but appear sequentially in order, \( \kappa_3 \leq \kappa_1 \).

Rationale for Time → Kime Extension

- Math – Time is a special case of kime, \( \kappa = |e^{i\theta}| \) where \( \kappa = 0 \) (no-phase)
  - additively a multiplicative (exponential) group, (multiplicative) unity/identity \( \kappa = 1 \)
  - multiplicative inverses, multiplicative identity, associativity \( \kappa(\kappa_1 \kappa_2 \kappa_3) = (\kappa_1 \kappa_2) \kappa_3 \)
  - The ring domain (\( \mathbb{R}^2 \)) and a complex-algebraic field (\( \mathbb{C} \))
    - Additive unity (0), element additive inverse (\( -1 \)) \( \frac{1}{\kappa} = \kappa_1 \) is an outside \( \mathbb{R}^2 \) (time-domain)
  - \( x^2 + 1 = 0 \) has no solutions in time (or in \( \mathbb{R}^2 \))...

- Classical time (\( \mathbb{R}^2 \)) is a positive cone over the field of the real numbers (\( \mathbb{R} \))
- Time forms a subgroup of the multiplicative group of the reals
- Whereas kime (\( \kappa \)) is an algebraically closed prime field that naturally extends time
- Time is ordered & kime is not – this cause magnitude preserves the intensional time order
- Kime (\( \kappa \)) represents the smallest natural extension of time, complete field that agrees with time
- The time group is closed under addition, multiplication, and division (but not subtraction), it has the topology of \( \mathbb{R} \) and the structure of a multiplicative topological group \( \mathbb{C} \) isomorphic to the group \( \mathbb{C} \).

Physics

- Did time start (20/2/2016) [DOI: 10.1007/978-3-319-58648-1]
- And C-I Hilbert space quantum theories make different predictions [DOI: 10.1056/01/2016-01-01]

A Spacekime Solution to Wave Equation

- Math Generalizations:
  - Derived other spacekime concepts: law of addition of velocities, energy-momentum conservation law, stability conditions for particles moving in spacekime, conditions for nonzero rest particle mass, causal structure of spacekime, and solutions of the ultrahyperbolic wave equation under Cauchy initial data...
(Many) Spacekime Open Math Problems

- **Eurality**
  - Let's look at particle velocities in the 4D Minkowski spacetime \((\mathbb{R}^4)\), a measure space where gas particles move spatially and evolve longitudinally in time. Let \(\mathbf{p} = \mu_p\) be a measure on \(\mathbb{R}^4\), \(\mathbf{f}(\mathbf{x}, t) \in L^1(\mathbb{R}^4)\) be an integrable function (e.g., velocity of a particle), and \(\mathbb{R}^1 \to \mathbb{R}^3\) be a measure-preserving transformation at position \(\mathbf{x} \in \mathbb{R}^3\) and time \(t \in \mathbb{R}^1\).

  - A pointwise ergodic theorem argues that in a measure theoretic sense, the average of \(f\) (e.g., velocity) over all particles in the gas system at a fixed time, \(f = \frac{1}{\mu_p(\mathbb{R}^4)} \int \mathbf{f}(\mathbf{x}, t) \cdot d\mu_p\), will be equal to the average of \(f\) of just one particle \((\mathbf{x})\) over the entire time span, \(f = \lim_{t \to \infty} \left( \sum_{n=1}^{\infty} \int f(\mathbf{t+n}, t) \cdot d\mu_p \right) = f\).

  - The space probability measure is denoted by \(\mu_p\) and the transformation \(T^i\) reprensents the dynamics (time evolution) of the particle starting with an initial spatial location \(T^i\).

  - Investigate the ergodic properties of various transformations in the 5D spacekime:
    \[ f = E_\phi(f) = \frac{1}{\mu_p(\mathbb{R}^5)} \int f(\mathbf{x}, t, \phi) \cdot d\mu_p = \lim_{t \to \infty} \left( \sum_{n=1}^{\infty} \int \mathbf{f}(\mathbf{t+n}, t, \phi) \cdot d\mu_p \right) = f \]

- **Data Science & AI**
  - A particle is a small localized object that permits observations and characterization in its physical or chemical properties.
  - A particle system is a collection of independent particles and observable characteristics, in a closed system.
  - A particle system is computable if (1) the system is hypothesis-consistent, complete, and (2) the unknown internal states of the system don't influence the computation (wavefunction, internal, probabilities, etc.).

- **Spacekime Analytics: fMRI Example**
  - 3D Isosurface Reconstruction of (2D space x 1D time) fMRI signal

- **Spacekime Analytics: Kime-series = Surfaces (not curves)**
  - In the 5D spacekime manifold, time-series curves extend to kime-series, i.e., surfaces parameterized by kime-magnitude (4) and the kime-phase (5).
  - Kime-phase aggregating operators that can be used to transform standard time-series curves to spacekime kime-surfaces, which can be modeled, interpreted, and predicted using advanced spacekime analytics.
Bayesian Inference Representation

- Suppose we have a single space-time observation \( X = \{x_i\} \sim p(x | \gamma) \) and \( \gamma \sim p(\gamma | \phi) \) is a process parameter (or vector) that we are trying to estimate.
- Spacekime analytics aims to make appropriate inference about the process \( X \).
- The sampling distribution, \( p(x | \gamma, \phi) \), is the distribution of the observed data \( X \) conditional on the parameter \( \gamma \) and the prior distribution, \( p(\gamma | \phi) \), of the parameter \( \gamma \) before the data \( X \) is observed, \( \phi = \text{phase aggregate} \).
- Assume that the hyperparameter vector \( \phi \), which represents the kime-phase estimates for the process, can be estimated by \( \phi = \phi' \).
- Such estimates may be obtained from an oracle (model distribution), approximated using similar datasets, acquired as phases from samples of analogous processes, derived via some phase-aggregation strategy, or computed via Laplace transform.
- Let the posterior distribution of the parameter \( \gamma \) given the observed data \( X = \{x_i\} \) be \( p(\gamma | X, \phi') \) and the process parameter distribution of the kime-phase hyperparameter vector \( \phi \) be \( p(\phi | X) \).

Bayesian Inference Simulation

Summary statistics for the original process (cohort \( A \)) and the corresponding values of their counterparts computed using the spacekime reconstructed signals based on kime phases of cohorts \( A, C \), and \( D \). The estimated for the latter three cohorts correspond to reconstructions using a single space-time observation (i.e., single kime-magnitude) and alternative kime phases (in this case, kime-phases derived from cohorts \( B, C \), and \( D \)).

<table>
<thead>
<tr>
<th>Spacetime</th>
<th>Spacekime Reconstructions (single kime-magnitude)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Summary</td>
<td>( \text{Spacekime} )</td>
</tr>
<tr>
<td></td>
<td>( \text{Original} )</td>
</tr>
<tr>
<td>Step 1</td>
<td>( -0.28508 )</td>
</tr>
<tr>
<td>Step 2</td>
<td>( 0.06000 )</td>
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<tr>
<td>Step 3</td>
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<tr>
<td>Step 4</td>
<td>( 0.51354 )</td>
</tr>
<tr>
<td>Step 5</td>
<td>( 0.60267 )</td>
</tr>
</tbody>
</table>

Bayesian Inference Simulation

The correlation between the original data (\( A \)) and its reconstruction using a single kime-magnitude and the correct kime-phases (\( C \)) is \( \rho(A, C) = 0.89 \).

The correlation between the original data (\( A \)) and its reconstruction using a single kime-magnitude and the incorrect kime-phases (\( D \)) is \( \rho(A, D) = 0.56 \).

This strong correlation suggests that a substantial part of the \( A \) process energy can be recovered using only a single observation. In this case, to reconstruct the signal back into spacetime and compute the corresponding correlation, we used a single kime-magnitude (sample-size=1) and process \( C \) kime-phases.
Bayesian Inference Simulation

Let's demonstrate the Bayesian inference corresponding to this spacekime data analytic problem using a simulated bimodal experiment:

\[ X = 0.3U + 0.7V, \text{ where } U \sim N(0,1) \text{ and } V \sim N(5,3) \]

Specifically, we will illustrate the Bayesian inference using repeated single spacetime observations from cohort A, \(X = \{x_i\}, \text{ and varying kime-phase priors (} \theta \text{ = phase aggregator) obtained from cohorts B, C, or D, using different posterior predictive distributions.} \)

Relations between the empirical data distribution (dark blue) and samples from the posterior predictive distribution, representing Bayesian simulated spacekime reconstructions (light blue). The derived Bayesian estimates do not perfectly match the empirical distribution of the simulated data, yet there is clearly information encoding that is captured by the spacekime data reconstructions.

This signal compression can be exploited by subsequent model-based or model-free data analytic strategies for retrospective prediction, prospective forecasting, ML classification, AI derived clustering, and other spacekime inference methods.

Spacekime Analytics: Demos

- Tutorials
  - [https://TCIU.predictive.space](https://TCIU.predictive.space)
  - [https://SpaceKime.org](https://SpaceKime.org)

- R Package
  - [https://cran.rstudio.com/web/packages/TCIU](https://cran.rstudio.com/web/packages/TCIU)

- GitHub
  - [https://github.com/SOCR/TCIU](https://github.com/SOCR/TCIU)

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Slides Online: "SOCR News"