Data Science, Time Complexity & Spacekime Analytics

Ivo D. Dinov

Statistics Online Computational Resource
Health Behavior & Biological Sciences
Computational Medicine & Bioinformatics
Neuroscience Graduate Program
Michigan Institute for Data Science
University of Michigan

https://SOCR.umich.edu

Joint work with Milen V. Velev (BTU)

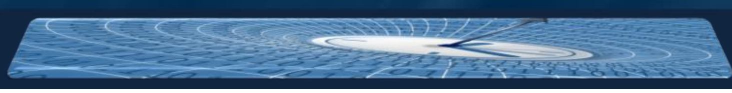
Based on an upcoming book "*Data Science: Time Complexity, Inferential Uncertainty & Spacekime Analytics*"

SCHOOL OF NURSING
STATISTICS ONLINE COMPUTATIONAL RESOURCE (SOCR)
UNIVERSITY OF MICHIGAN

Slides Online:
"SOCR News"

Outline

- ❑ Motivation: Big Data Analytics Challenges
- ❑ Complex-Time (*kime*)
- ❑ Spacekime Calculus & Math Foundations
- ❑ Open Spacekime Problems
- ❑ Neuroscience Applications
  - ❑ Longitudinal Neuroimaging (UKBB, fMRI)



Big Data Analytics Challenges

Common Characteristics of Big Data

IBM Big Data 4V's: Volume, Variety, Velocity & Veracity

Big Bio Data Dimensions	Tools
Size	Harvesting and management of vast amounts of data
Complexity	Wranglers for dealing with heterogeneous data
Incongruency	Tools for data harmonization and aggregation
Multi-source	Transfer and joint multivariate representation & modeling
Multi-scale	Macro → meso → micro → nano scale observations
Time	Techniques accounting for longitudinal effects (e.g., time corr)
Incomplete	Reliable management of missing data, imputation

Example: analyzing observational data of 1,000's Parkinson's disease patients based on 10,000's signature biomarkers derived from multi-source imaging, genetics, clinical, physiologic, phenomics and demographic data elements

Software developments, student training, service platforms and methodological advances associated with the Big Data Discovery Science all present existing opportunities for learners, educators, researchers, practitioners and policy makers

Dinov, *GigaScience* (2016)

Gao *et al., SciRep* (2018)

Complex-Time (*kime*)
&
Spacekime Foundations

The Fourier Transform

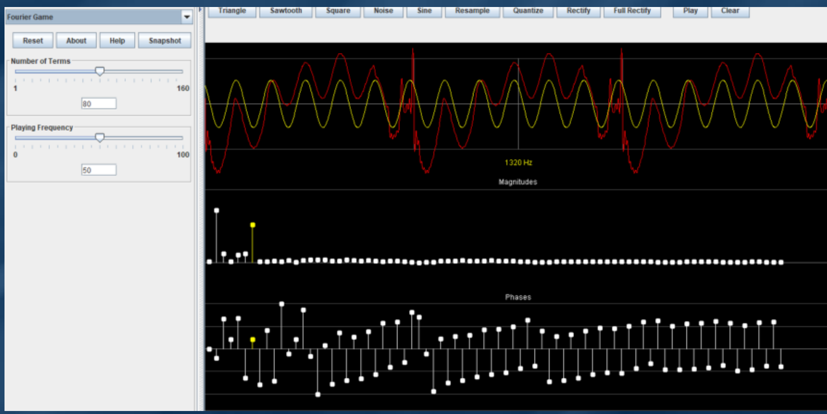
By separability, the classical **spacetime Fourier transform** is just four Fourier transforms, one for each of the four spacetime dimensions, $(\boldsymbol{x}, t) = (x, y, z, t)$. The FT is a function of the angular frequency ω that propagates in the wave number direction $\boldsymbol{k}$ (space frequency). Symbolically, the forward and inverse Fourier transforms of a 4D ($n = 4$) spacetime function f, are defined by:

$$FT(f) = \hat{f}(\boldsymbol{k}, \omega) = \frac{1}{(2\pi)^{\frac{n}{2}}} \int f(\boldsymbol{x}, t) e^{i(\omega t - \boldsymbol{kx})} dt d^3\boldsymbol{x},$$

$$IFT(\hat{f}) = \hat{\hat{f}}(\boldsymbol{x}, t) = \frac{1}{(2\pi)^{\frac{n}{2}}} \int \hat{f}(\boldsymbol{k}, \omega) e^{-i(\omega t - \boldsymbol{kx})} d\omega d^3\boldsymbol{k}.$$

$$\left[\hat{\hat{f}}(\boldsymbol{x}, t) = IFT(\hat{f}) = IFT(FT(f)) = f(\boldsymbol{x}, t), \qquad \forall z \in \mathbb{C}, z = \underbrace{A}_{mag} e^{i \overbrace{\varphi}^{phase}} \right]$$

1D Fourier Transform Example



SOCR 1D Fourier / Wavelet signal decomposition into *magnitudes* and *phases (Java applet)*

Top-panel: original signal (image), white-color curve drawn manually by the user and the reconstructed synthesized (IFT) signal, red-color curve, computed using the user modified magnitudes and phases

Bottom-panels: the Fourier analyzed signal (FT) with its magnitudes and phases

http://www.socr.ucla.edu/htmls/game/Fourier_Game.html (Java Applet)

2D Fourier Transform – The Importance of Magnitudes & Phases

Fourier Analysis

(real part of the Forward Fourier Transform)

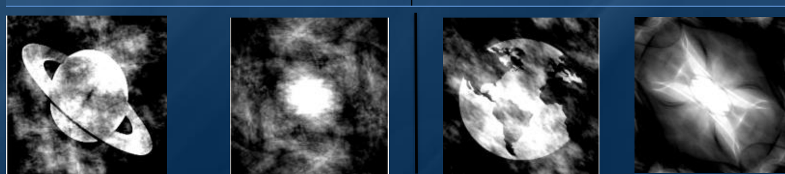
Earth			Saturn		

2D image 1 (Earth)	Magnitude FT(Earth)	Phase FT(Earth)	2D image 2 (**Saturn**)	Magnitude FT(**Saturn**)	Phase FT(**Saturn**)

Fourier Synthesis

(real part of the Inverse Fourier Transform)

Earth		Saturn	

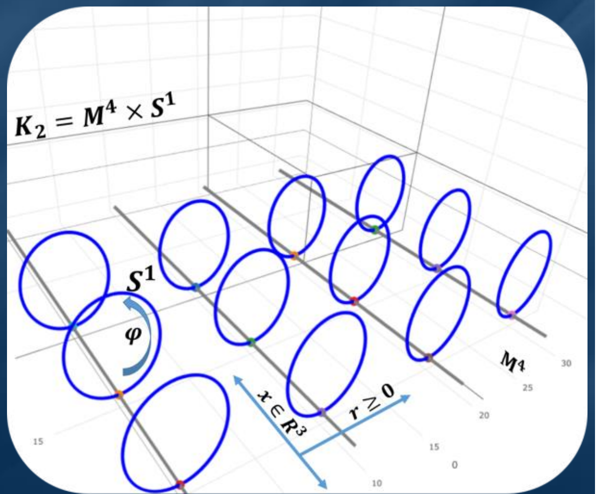


IFT using Earth-magnitude & Saturn-phase	IFT using Earth-magnitude & nil-phase	IFT using Saturn-magnitude & Earth-phase	IFT using Saturn-magnitude & nil-phase

Kaluza-Klein Theory

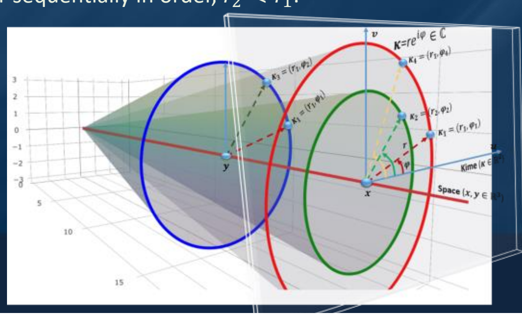
- Theodor Kaluza (1921) developed a math extension of the classical general relativity theory to 5D. This included the metric, the field equations, the equations of motion, the stress-energy tensor, and the cylinder condition. Physicist Oskar Klein (1926) interpreted Kaluza's 3D+2D theory in quantum mechanical space and proposed that the fifth dimension was curled up and microscopic.

- The topology of the 5D Kaluza-Klein spacetime is $K_2 \cong M^4 \times S^1$, where M^4 is a 4D Minkowski spacetime and S^1 is a circle (non-traversable).

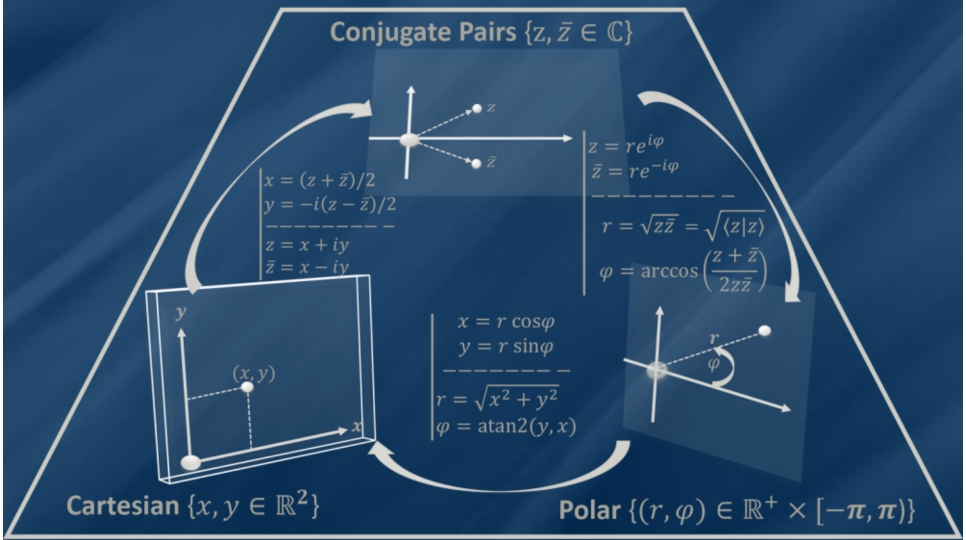


Complex-Time (*Kime*)

- At a given spatial location, $\boldsymbol{x}$, complex time (*kime*) is defined by $\kappa = re^{i\varphi} \in \mathbb{C}$, where:
  - the magnitude represents the longitudinal events order ($r > 0$) and characterizes the longitudinal displacement in time, and
  - event phase ($-\pi \leq \varphi < \pi$) is an angular displacement, or event direction
- There are multiple alternative parametrizations of kime in the complex plane
- Space-kime manifold is $\mathbb{R}^3 \times \mathbb{C}$:
  - $(\boldsymbol{x}, k_1)$ and $(\boldsymbol{x}, k_4)$ have the same spacetime representation, but different spacekime coordinates,
  - $(\boldsymbol{x}, k_1)$ and $(\boldsymbol{y}, k_1)$ share the same kime, but represent different spatial locations,
  - $(\boldsymbol{x}, k_2)$ and $(\boldsymbol{x}, k_3)$ have the same spatial-locations and kime-directions, but appear sequentially in order, $r_2 < r_1$.



Kime Parameterizations



The Spacekime Manifold

- **Spacekime:** $(\boldsymbol{x}, \boldsymbol{k}) = \left(\underbrace{x^1, x^2, x^3}_{\text{space}}, \underbrace{c\kappa_1 = x^4, c\kappa_2 = x^5}_{\text{kime}}\right) \in X, \quad c \sim 3 \times 10^8\ m/s$
- **Kevents** (*complex events*): points (or states) in the spacekime manifold X. Each kevent is defined by where ($\boldsymbol{x} = (x, y, z)$) it occurs in space, what is its *causal longitudinal order* $\left(r = \sqrt{(x^4)^2 + (x^5)^2}\right)$, and in what *kime-direction* ($\varphi = \text{atan2}(x^5, x^4)$) it takes place.
- **Spacekime interval** (ds) is defined using the general Minkowski 5×5 metric tensor $\left(\lambda_{ij}\right)_{i=1,j=1}^{5,5}$, which characterizes the geometry of the *(generally curved)* *spacekime* manifold:

$$ds^2 = \sum_{i=1}^{5}\sum_{j=1}^{5} \lambda_{ij} dx^i dx^j = \lambda_{ij} dx^i dx^j$$

- *Euclidean (flat) spacekime* metric corresponds to the tensor:

$$(\lambda_{ij}) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

  - Spacelike intervals correspond to $ds^2 > 0$, where an inertial frame can be found such that two kevents $a, b \in X$ are simultaneous. An object can't be present at two kevents which are separated by a spacelike interval.
  - Lightlike intervals correspond to $ds^2 = 0$. If two kevents are on the line of a photon, then they are separated by a lightlike interval and a ray of light could travel between the two kevents.
  - Kimelike intervals correspond to $ds^2 < 0$. An object can be present at two different kevents, which are separated by a kimelike interval.

Spacekime Calculus

- Kime **Wirtinger derivative** (first order kime-derivative at $\boldsymbol{k} = (r, \varphi)$):

$$f'(z) = \frac{\partial f(z)}{\partial z} = \frac{1}{2}\left(\frac{\partial f}{\partial x} - i\frac{\partial f}{\partial y}\right) \text{ and } f'(\bar{z}) = \frac{\partial f(\bar{z})}{\partial \bar{z}} = \frac{1}{2}\left(\frac{\partial f}{\partial x} + i\frac{\partial f}{\partial y}\right).$$

In Conjugate-pair basis: $df = \partial f + \bar{\partial} f = \frac{\partial f}{\partial z} dz + \frac{\partial}{\partial \bar{z}} d\bar{z}$

In Polar kime coordinates:

$$f'(k) = \frac{\partial f(k)}{\partial k} = \frac{1}{2}\left(\cos\varphi \frac{\partial f}{\partial r} - \frac{1}{r}\sin\varphi \frac{\partial f}{\partial \varphi} - i\left(\sin\varphi \frac{\partial f}{\partial r} + \frac{1}{r}\cos\varphi \frac{\partial f}{\partial \varphi}\right)\right) = \frac{e^{-i\varphi}}{2}\left(\frac{\partial f}{\partial r} - \frac{i}{r}\frac{\partial f}{\partial \varphi}\right)$$

$$f'(\bar{k}) = \frac{\partial f(\bar{k})}{\partial \bar{k}} = \frac{1}{2}\left(\cos\varphi \frac{\partial f}{\partial r} - \frac{1}{r}\sin\varphi \frac{\partial f}{\partial \varphi} + i\left(\sin\varphi \frac{\partial f}{\partial r} + \frac{1}{r}\cos\varphi \frac{\partial f}{\partial \varphi}\right)\right) = \frac{e^{i\varphi}}{2}\left(\frac{\partial f}{\partial r} + \frac{i}{r}\frac{\partial f}{\partial \varphi}\right)$$

- Kime **Wirtinger acceleration** (second order kime-derivative at $\boldsymbol{k} = (r, \varphi)$):

$$f''(\boldsymbol{k}) = \frac{1}{4r^2}\left((\cos\varphi - i\sin\varphi)^2\left(2i\frac{\partial f}{\partial \varphi} - \frac{\partial^2 f}{\partial \varphi^2} + r\left(-\frac{\partial f}{\partial r} - 2i\frac{\partial^2 f}{\partial r \partial \varphi} + r\frac{\partial^2 f}{\partial r^2}\right)\right)\right).$$

Spacekime Calculus

- Kime **Wirtinger integration**:

The *path-integral* of a complex function $f: \mathbb{C} \to \mathbb{C}$ on a specific path connecting $z_a \in \mathbb{C}$ to $z_b \in \mathbb{C}$ is defined by generalizing Riemann sums:

$$\lim_{|z_{i+1}-z_i| \to 0} \sum_{i=1}^{n-1} (f(z_i)(z_{i+1} - z_i)) \cong \oint_{z_a}^{z_b} f(z_i) dz.$$

This assumes the path is a polygonal arc joining z_a to z_b, via $z_1 = z_a, z_2, z_3, \ldots, z_n = z_b$, and we integrate the piecewise constant function $f(z_i)$ on the arc joining $z_i \to z_{i+1}$. *Assumptions*: the path $z_a \to z_b$ needs to be defined and the limit of the generalized Riemann sums, as $n \to \infty$, will yield a complex number representing the Wirtinger integral of the function over the path.

- Similarly, extend the classical area integrals, indefinite integral, and Laplacian:

Definite area integral: for $\Omega \subseteq \mathbb{C}$, $\int_\Omega f(z) dz d\bar{z}$.

Indefinite integral: $\int f(z) dz d\bar{z}$, $df = \frac{\partial f}{\partial z} dz + \frac{\partial f}{\partial \bar{z}} d\bar{z}$.

The *Laplacian* in terms of conjugate pair coordinates is $\Delta f = \nabla^2 f = 4\frac{\partial f}{dz}\frac{\partial f}{d\bar{z}} = 4\frac{\partial f}{d\bar{z}}\frac{\partial f}{dz}$.

Newton's equations of motion in kime

$$\left|\begin{array}{l} v = at + v_o \\ x = x_o + v_o t + \frac{1}{2}at^2 \\ v^2 = 2a(x - x_o) + v_o^2 \end{array}\right. \Rightarrow \left|\begin{array}{l} v \quad = a_1 k_1 + v_{o1} = a_2 k_2 + v_{o2}\,, \\ x = x_{o1} + v_{o1}k_1 + \frac{1}{2}a_1 k_1^2 = x_{o2} + v_{o2}k_2 + \frac{1}{2}a_2 k_2^2, \\ \sqrt{v_2^4 - v^2 v_2^2} = -a_1\left(x - x_{o1}\right) + \sqrt{v_{o2}^4 - v_o^2 v_{o2}^2}, \\ \sqrt{v_1^4 - v^2 v_1^2} = -a_2\left(x - x_{o2}\right) + \sqrt{v_{o1}^4 - v_o^2 v_{o1}^2} \end{array}\right.$$

- Derived from the Kime Wirtinger velocity and acceleration
  - Kime-velocity ($\boldsymbol{k} = (t, \varphi)$) is defined by the Wirtinger derivative of the position with respect to kime:

$$v(\boldsymbol{k}) = \frac{\partial \boldsymbol{x}}{\partial \boldsymbol{k}} = \frac{1}{2}\left(\cos\varphi \frac{\partial \boldsymbol{x}}{\partial t} - \frac{1}{t}\sin\varphi \frac{\partial \boldsymbol{x}}{\partial \varphi} - i\left(\sin\varphi \frac{\partial \boldsymbol{x}}{\partial t} + \frac{1}{t}\cos\varphi \frac{\partial \boldsymbol{x}}{\partial \varphi}\right)\right)$$

  - The directional kime derivatives v_1 and v_2, (**e** = unit vector of spatial directional change):

$$\boldsymbol{v}_1 = \frac{\sqrt{dx^2+dy^2+dz^2}}{dk_1}\boldsymbol{e} = \frac{\sqrt{dx^2+dy^2+dz^2}}{\cos\varphi dt - t\sin\varphi d\varphi}\boldsymbol{e}, \quad \boldsymbol{v}_2 = \frac{\sqrt{dx^2+dy^2+dz^2}}{dk_2}\boldsymbol{e} = \frac{\sqrt{dx^2+dy^2+dz^2}}{\sin\varphi dt + t\cos\varphi d\varphi}\boldsymbol{e}$$

Spacekime Generalizations

- Spacekime generalization of **Lorentz transform** between two reference frames, K & K':

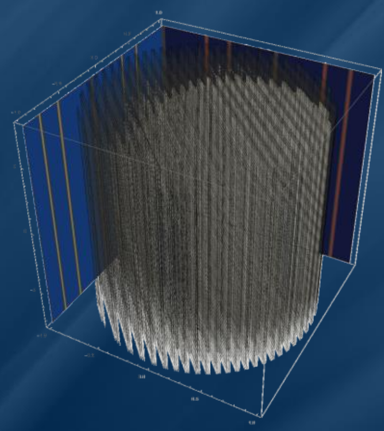
(the interval ds is Lorentz transform invariant)

$$\underbrace{\begin{pmatrix} x' \\ y' \\ z' \\ k_1' \\ k_2' \end{pmatrix}}_{\in K'} = \begin{pmatrix} \zeta & 0 & 0 & -\frac{c^2}{v_1}\beta^2\zeta & -\frac{c^2}{v_2}\beta^2\zeta \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ -\frac{1}{v_1}\beta^2\zeta & 0 & 0 & 1 + (\zeta - 1)\frac{c^2}{(v_1)^2}\beta^2 & (\zeta - 1)\frac{c^2}{v_1 v_2}\beta^2 \\ -\frac{1}{v_2}\beta^2\zeta & 0 & 0 & (\zeta - 1)\frac{c^2}{v_1 v_2}\beta^2 & 1 + (\zeta - 1)\frac{c^2}{(v_2)^2}\beta^2 \end{pmatrix} \underbrace{\begin{pmatrix} x \\ y \\ z \\ k_1 \\ k_2 \end{pmatrix}}_{\in K}$$

where $0 \le \beta = \frac{1}{\sqrt{\left(\frac{c}{v_1}\right)^2 + \left(\frac{c}{v_2}\right)^2}} \le 1$ & $\zeta = \frac{1}{\sqrt{1-\beta^2}} \ge 1$.

Spacekime Solution to Wave Equation

- ❑ Math Generalizations
- ❑ Derived other spacekime concepts: law of addition of velocities, energy-momentum conservation law, stability conditions for particles moving in spacekime, conditions for nonzero rest particle mass, causal structure of spacekime, and solutions of the ultrahyperbolic wave equation under Cauchy initial data …



Wang et al., 2021 | Dinov & Velev (2021)

Ultrahyperbolic Wave Equation – Cauchy Initial Data

- ❑ **Nonlocal constraints** yield the existence, uniqueness & stability of local and global solutions to the ultrahyperbolic wave equation under Cauchy initial data …

$$\underbrace{\sum_{i=1}^{d_s} \partial_{x_i}^2 u \equiv \Delta_{\boldsymbol{x}} u(\boldsymbol{x},\boldsymbol{\kappa})}_{\text{spatial Laplacian}} = \underbrace{\Delta_{\boldsymbol{\kappa}} u(\boldsymbol{x},\boldsymbol{\kappa}) \equiv \sum_{i=1}^{d_t} \partial_{\kappa_i}^2 u}_{\text{temporal Laplacian}}, \quad \underbrace{\left| \begin{array}{l} u_o = u\Big(\underbrace{\boldsymbol{x}}_{\boldsymbol{x}\in D_s}, \underbrace{0,\boldsymbol{\kappa}_{-1}}_{\boldsymbol{\kappa}\in D_t}\Big) = f(\boldsymbol{x},\boldsymbol{\kappa}_{-1}) \\ u_1 = \partial_{\kappa_1} u(\boldsymbol{x},0,\boldsymbol{\kappa}_{-1}) = g(\boldsymbol{x},\boldsymbol{\kappa}_{-1}) \end{array} \right.}_{\text{initial conditions (Cauchy Data)}}$$

where $\boldsymbol{x} = (x_1, x_2, \dots, x_{d_s}) \in \mathbb{R}^{d_s}$ and $\boldsymbol{\kappa} = (\kappa_1, \kappa_2, \dots, \kappa_{d_t}) \in \mathbb{R}^{d_t}$ are the Cartesian coordinates in the d_s space and d_t time dims.

Stable local solution over a Fourier frequency region defined by $|\boldsymbol{\xi}| \geq |\boldsymbol{\eta}_{-1}|$ … nonlocal constraints:

$$\hat{u}\Big(\boldsymbol{\xi}, \underbrace{\kappa_1, \boldsymbol{\eta}_{-1}}_{\boldsymbol{\eta}}\Big) = \cos\left(2\pi\, \kappa_1 \sqrt{|\boldsymbol{\xi}|^2 - |\boldsymbol{\eta}_{-1}|^2}\right) \underbrace{\hat{u}_o(\boldsymbol{\xi}, \boldsymbol{\eta}_{-1})}_{c_1} + \sin\left(2\pi\, \kappa_1 \sqrt{|\boldsymbol{\xi}|^2 - |\boldsymbol{\eta}_{-1}|^2}\right) \underbrace{\frac{\hat{u}_1(\boldsymbol{\xi}, \boldsymbol{\eta}_{-1})}{2\pi \sqrt{|\boldsymbol{\xi}|^2 - |\boldsymbol{\eta}_{-1}|^2}}}_{c_2},$$

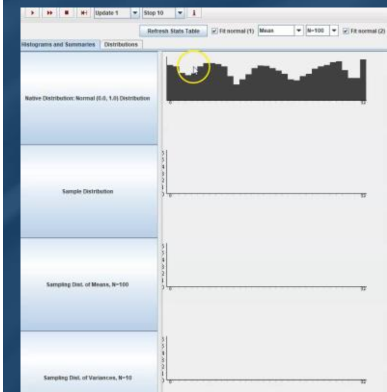
where $\mathcal{F}\begin{pmatrix} u_o \\ u_1 \end{pmatrix} = \begin{pmatrix} \hat{u}_o \\ \hat{u}_1 \end{pmatrix} = \begin{pmatrix} \hat{u}_o(\boldsymbol{\xi}, \boldsymbol{\eta}_{-1}) \\ \hat{u}_1(\boldsymbol{\xi}, \boldsymbol{\eta}_{-1}) \end{pmatrix} = \begin{pmatrix} \hat{u}(\boldsymbol{\xi}, \boldsymbol{\eta}_{-1}) \\ \partial_{\kappa_1} \hat{u}(\boldsymbol{\xi}, \boldsymbol{\eta}_{-1}) \end{pmatrix}$.

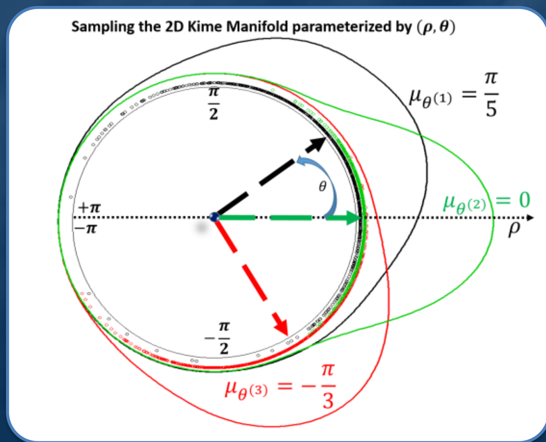
$$u\Big(\boldsymbol{x}, \underbrace{\kappa_1, \boldsymbol{\kappa}_{-1}}_{\boldsymbol{\kappa}}\Big) = \mathcal{F}^{-1}(\hat{u})(\boldsymbol{x},\boldsymbol{\kappa}) = \int_{\hat{D}_s \times \hat{D}_{t_{-1}}} \hat{u}(\boldsymbol{\xi}, \kappa_1, \boldsymbol{\eta}_{-1}) \times e^{2\pi i \langle \boldsymbol{x}, \boldsymbol{\xi} \rangle} \times e^{2\pi i \langle \boldsymbol{\kappa}_{-1}, \boldsymbol{\eta}_{-1} \rangle} d\boldsymbol{\xi}\, d\boldsymbol{\eta}_{-1}.$$

Wang et al., 2021 | Dinov & Velev (2021)

Hidden Variable Theory & Random Sampling

- ❑ Kime phase distributions are mostly symmetric, random observations ≡ phase sampling

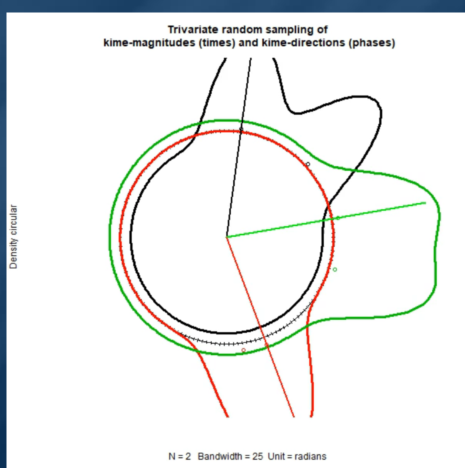




http://wiki.stat.ucla.edu/socr/index.php/SOCR_EduMaterials_Activities_GeneralCentralLimitTheorem

Dinov, Christou & Sanchez (2008) Dinov & Velev (2021)

Kime-Phase Sampling Simulation



https://Spacekime.org Dinov & Velev (2021)

(Many) Spacekime Open Math Problems

- **Ergodicity**

Let's look at the particle velocities in the 4D Minkowski spacetime (X), a measure space where gas particles move spatially and evolve longitudinally in time. Let $\mu = \mu_x$ be a measure on X, $\underline{f(\boldsymbol{x},t)} \in L^1(X,\mu)$ be an integrable function (e.g., velocity of a particle), and $\underline{T}: X \to X$ be a measure-preserving transformation at position $\boldsymbol{x} \in \mathbb{R}^3$ at time $t \in \mathbb{R}^+$.

Prove a pointwise ergodic theorem arguing that in a measure theoretic sense, the average of f over all particles in the gas system at a fixed time, $\bar{f} = E_t(f) = \int_{\mathbb{R}^3} f(\boldsymbol{x},t)d\mu_x$, will be equal to the average velocity (f) of just one particle ($\boldsymbol{x}$) over the entire time span,

$$\tilde{f} = \lim_{n\to\infty}\left(\frac{1}{n}\sum_{i=0}^{n} f(T^i\boldsymbol{x})\right).$$ That is, prove that $\bar{f} \equiv \tilde{f}$.

The spatial probability measure is denoted by μ_x and the transformation $T^i\boldsymbol{x}$ represents the dynamics (time evolution) of the particle starting with an initial spatial location $T^o\boldsymbol{x} = \boldsymbol{x}$.

Investigate the ergodic properties of various transformations in the 5D **spacekime**:

$$\underbrace{\bar{f} \equiv E_\kappa(f) = \frac{1}{\mu_x(X)}\int f\left(\boldsymbol{x}, \underbrace{t,\phi}_{\kappa}\right)d\mu_x}_{\text{space averaging}} \stackrel{?}{=} \underbrace{\lim_{t\to\infty}\left(\frac{1}{t}\sum_{i=0}^{t}\left(\int_{-\pi}^{+\pi} f(T^i\boldsymbol{x},t,\phi)d\Phi\right)\right)}_{\text{kime averaging}} \equiv \tilde{f}$$

Spacekime Connection to Data Science & Neuroscience?

Mathematical-Physics ⟹ Data/Neuro Sciences

Mathematical-Physics	Data/Neuro Sciences
A **particle** is a small localized object that permits observations and characterization of its physical or chemical properties	An **object** is something that exists by itself, actually or potentially, concretely or abstractly, physically or incorporeal (e.g., person, subject, etc.)
An **observable** a dynamic variable about particles that can be measured	A **feature** is a dynamic variable or an attribute about an object that can be measured
Particle **state** is an observable particle characteristic (e.g., position, momentum)	**Datum** is an observed quantitative or qualitative value, an instantiation, of a feature
Particle **system** is a collection of independent particles and observable characteristics, in a closed system	**Problem**, aka Data System, is a collection of independent objects and features, without necessarily being associated with apriori hypotheses
Wave-function	**Inference-function**
Reference-Frame **transforms** (e.g., Lorentz)	Data **transformations** (e.g., wrangling, log-transform)
State of a system is an observed measurement of all particles ~ wavefunction	**Dataset (data)** is an observed instance of a set of datum elements about the problem system, $\boldsymbol{O} = \{\boldsymbol{X}, \boldsymbol{Y}\}$
A **particle system is computable** if (1) the entire system is logical, consistent, complete and (2) the unknown internal states of the system don't influence the computation (wavefunction, intervals, probabilities, etc.)	**Computable data object** is a very special representation of a dataset which allows direct application of computational processing, modeling, analytics, or inference based on the observed dataset
…	…

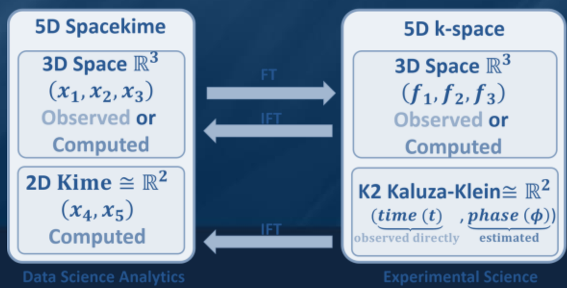
Mathematical-Physics ⟹ Data Science

Math-Physics	Data Science
Wavefunction Wave equ problem: $\left(\frac{\partial^2}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2}{\partial t}\right)\psi(x,t) = 0$ Complex Solution: $\psi(x,t) = Ae^{i(kx-wt)}$ where $\left\lvert\frac{w}{k}\right\rvert = v$, represents a traveling wave	*Inference function* - describing a solution to a specific data analytic system (a problem). For example, • A linear (GLM) model represents a solution of a prediction inference problem, $\boldsymbol{Y} = \boldsymbol{X}\beta$, where the inference function quantifies the effects of all independent features ($\boldsymbol{X}$) on the dependent outcome ($\boldsymbol{Y}$), data: $\boldsymbol{O} = \{\boldsymbol{X}, \boldsymbol{Y}\}$: $\psi(\boldsymbol{O}) = \psi(\boldsymbol{X},\boldsymbol{Y}) \Rightarrow \hat{\beta} = \hat{\beta}^{OLS} = \langle X\|X\rangle^{-1}\langle X\|Y\rangle = \left(X^T X\right)^{-1} X^T Y.$ • A non-parametric, non-linear, alternative inference is SVM classification. If $\boldsymbol{\psi_x} \in \boldsymbol{H}$, is the lifting function $\boldsymbol{\psi}: \boldsymbol{R^\eta} \to \boldsymbol{R^d}$ ($\boldsymbol{\psi}: \boldsymbol{x} \in \boldsymbol{R^\eta} \to \tilde{x} = \boldsymbol{\psi_x} \in \boldsymbol{H}$), where $\boldsymbol{\eta} \ll \boldsymbol{d}$, the kernel $\boldsymbol{\psi_x(y)} = \langle \boldsymbol{x}\|\boldsymbol{y}\rangle: \boldsymbol{O} \times \boldsymbol{O} \to \boldsymbol{R}$ transformes non-linear to linear separation, the observed data $\boldsymbol{O_i} = \{\boldsymbol{x_i}, \boldsymbol{y_i}\} \in \boldsymbol{R^\eta}$ are lifted to $\boldsymbol{\psi_{O_i}} \in \boldsymbol{H}$. Then, the SVM prediction operator is the weighted sum of the kernel functions at $\boldsymbol{\psi_{O_i}}$, where $\boldsymbol{\beta^*}$ is a solution to the SVM regularized optimization: $\langle \psi_O \| \beta^* \rangle_H = \sum_{i=1}^{n} p_i^* \langle \psi_O \| \psi_{O_i} \rangle_H$ The linear coefficients, $\boldsymbol{p_i^*}$, are the dual weights that are multiplied by the label corresponding to each training instance, $\{\boldsymbol{y_i}\}$. Inference always depends on the (input) data; however, it does not have 1-1 and onto bijective correspondence with the data, since the inference function quantifies predictions in a probabilistic sense.

GLM/SVM: https://DSPA.predictive.space \| Dinov, Springer (2018)

Spacekime Analytics

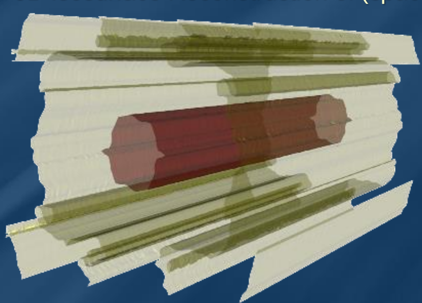
- Let's assume that we have:
  (1) Kime extension of Time, and
  (2) Parallels between wavefunctions ↔ inference functions
- Often, we can't directly observe (record) data natively in 5D spacekime.
- Yet, we can measure quite accurately the kime-magnitudes (r) as event orders, "times".
- To reconstruct the 2D spatial structure of kime, borrow tricks used by crystallographers [1] to resolve the structure of atomic particles by only observing the magnitudes of the diffraction pattern in k-space. This approach heavily relies on (1) <u>prior information</u> about the kime directional orientation (that may be obtained from using similar datasets and phase-aggregator analytical strategies), or (2) <u>experimental reproducibility</u> by repeated confirmations of the data analytic results using longitudinal datasets.

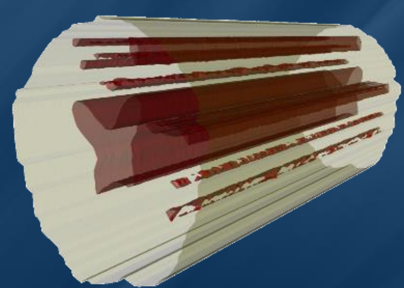


[1] Rodriguez, Ivanova, Nature 2015

Spacekime Analytics: fMRI Example

- 3D Isosurface Reconstruction of (space=2, time=1) fMRI signal





<u>4D spacetime</u>: Reconstruction using trivial phase-angle; kime=time=(magnitude, 0)

<u>5D Spacekime</u>: Reconstruction using correct kime=(magnitude, phase)

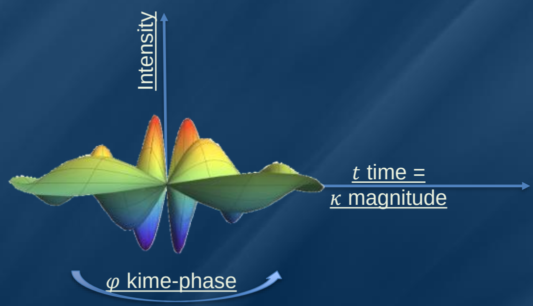
3D pseudo-spacetime reconstruction:

$$f = \hat{h}\,(\underbrace{x_1, x_2}_{space}, \underbrace{t}_{time})$$

Spacekime Analytics: Kime-series = Surfaces (not curves)

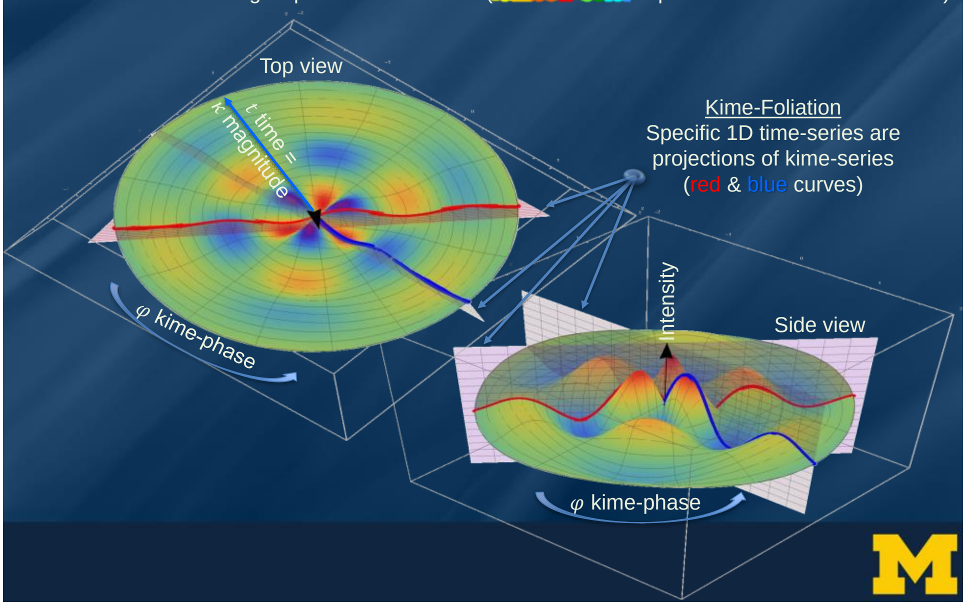
In the 5D spacekime manifold, time-series curves extend to kime-series, i.e., surfaces parameterized by kime-magnitude (t) and the kime-phase (φ).

Kime-phase aggregating operators that can be used to transform standard time-series curves to spacekime kime-surfaces, which can be modeled, interpreted, and predicted using advanced spacekime analytics.

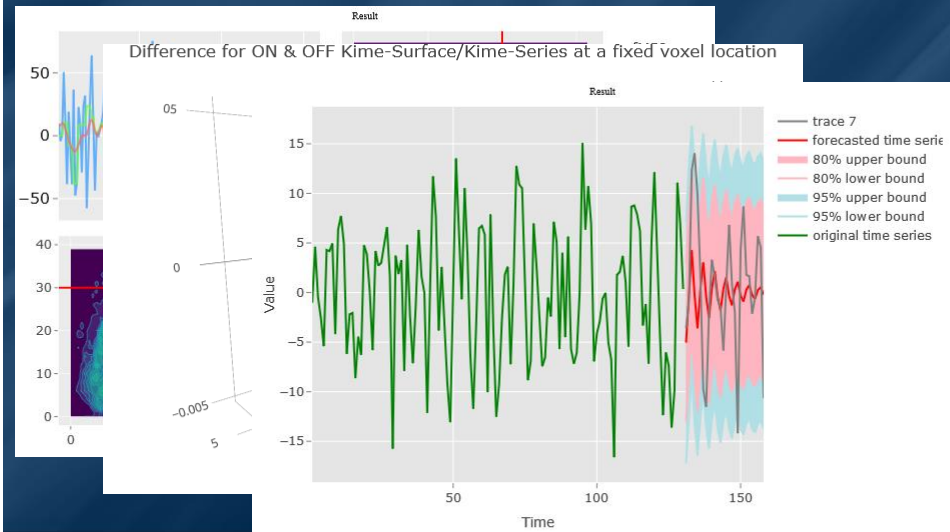


Spacekime Analytics: fMRI kime-series

fMRI *kime-series* at a single spatial voxel location (rainbow color represents fMRI kime intensities)



Spacetime Time-series ➔ *Spacekime Kime-surfaces*



Spacekime Analytics: **Demos**

- Tutorials
  - https://TCIU.predictive.space
  - https://SpaceKime.org
- R Package
  - https://cran.rstudio.com/web/packages/TCIU
- GitHub
  - https://github.com/SOCR/TCIU

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Slides Online:
“SOCR News”

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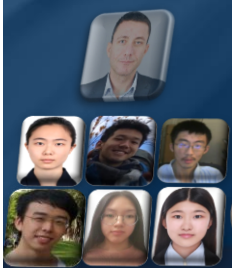
Collaborators

- SOCR: **Milen Velev, Yueyang Shen, Daxuan Deng, Zijing Li, Yongkai Qiu, Zhe Yin, Yufei Yang, Yuxin Wang, Rongqian Zhang, Yuyao Liu, Yupeng Zhang, Yunjie Guo**
- **UMich MIDAS/MNORC/AD/PD Centers**: Chuck Burant, Kayvan Najarian, Stephen Goutman, Stephen Strobbe, Hiroko Dodge, Chris Monk, Issam El Naqa, HV Jagadish, Brian Athey



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