Data Science, Time Complexity & Spacekime Analytics

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Joint work with Milen V. Velev (BTU)

Based on an upcoming book “Data Science: Time Complexity, Inferential Uncertainty & Spacekime Analytics”

Outline

- Motivation: Big Data Analytics Challenges
- Complex-Time (*kime*)
- Spacekime Calculus & Math Foundations
- Open Spacekime Problems
- Neuroscience Applications
  - Longitudinal Neuroimaging (UKBB, fMRI)
Big Data Analytics Challenges

Common Characteristics of Big Data

<table>
<thead>
<tr>
<th>Big Bio Data Dimensions</th>
<th>Tools</th>
<th>Example: analyzing observational data of 1,000's Parkinson's disease patients based on 10,000's signature biomarkers derived from multi-source imaging, genetics, clinical, physiologic, phenomics and demographic data elements</th>
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<tbody>
<tr>
<td>Size</td>
<td>Harvesting and management of vast amounts of data</td>
<td>Software developments, student training, service platforms and methodological advances associated with the Big Data Discovery Science all present existing opportunities for learners, educators, researchers, practitioners and policy makers</td>
</tr>
<tr>
<td>Complexity</td>
<td>Wranglers for dealing with heterogeneous data</td>
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<tr>
<td>Incongruency</td>
<td>Tools for data harmonization and aggregation</td>
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<td>Multi-source</td>
<td>Transfer and joint multivariate representation &amp; modeling</td>
<td></td>
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<tr>
<td>Multi-scale</td>
<td>Macro $\rightarrow$ meso $\rightarrow$ micro $\rightarrow$ nano scale observations</td>
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<tr>
<td>Time</td>
<td>Techniques accounting for longitudinal effects (e.g., time corr)</td>
<td></td>
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<tr>
<td>Incomplete</td>
<td>Reliable management of missing data, imputation</td>
<td></td>
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Dinov, GigaScience (2016)  
Complex-Time (*kime*)
&
Spacekime Foundations

The Fourier Transform

By separability, the classical spacetime Fourier transform is just four Fourier transforms, one for each of the four spacetime dimensions, \((x, t) = (x, y, z, t)\). The FT is a function of the angular frequency \(\omega\) that propagates in the wave number direction \(\mathbf{k}\) (space frequency). Symbolically, the forward and inverse Fourier transforms of a 4D \((n = 4)\) spacetime function \(f\), are defined by:

\[
FT(f) = \hat{f}(\mathbf{k}, \omega) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int f(x, t) e^{i(\omega t - \mathbf{k} \cdot \mathbf{x})} dt d^3x,
\]

\[
IFT(\hat{f}) = \hat{x}(x, t) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int \hat{f}(\mathbf{k}, \omega) e^{-i(\omega t - \mathbf{k} \cdot \mathbf{x})} d\omega d^3k.
\]

\[
\hat{x}(x, t) = IFT(\hat{f}) = IFT(FT(f)) = f(x, t), \quad \forall z \in \mathbb{C}, Z = \frac{A}{\text{mag}} e^{i \frac{\varphi}{\text{phase}}}
\]
1D Fourier Transform Example

SOCR 1D Fourier / Wavelet signal decomposition into magnitudes and phases (Java applet)

Top-panel: original signal (image), white-color curve drawn manually by the user and the reconstructed synthesized (IFT) signal, red-color curve, computed using the user modified magnitudes and phases

Bottom-panels: the Fourier analyzed signal (FT) with its magnitudes and phases

http://www.socr.ucla.edu/htmls/game/Fourier_Game.html (Java Applet)

2D Fourier Transform – The Importance of Magnitudes & Phases

Fourier Analysis
(real part of the Forward Fourier Transform)

<table>
<thead>
<tr>
<th></th>
<th>Earth</th>
<th>Saturn</th>
</tr>
</thead>
<tbody>
<tr>
<td>2D image 1 (Earth)</td>
<td>Magnitude FT(Earth)</td>
<td>Phase FT(Earth)</td>
</tr>
<tr>
<td>2D image 2 (Saturn)</td>
<td>Magnitude FT(Saturn)</td>
<td>Phase FT(Saturn)</td>
</tr>
</tbody>
</table>

Fourier Synthesis
(real part of the Inverse Fourier Transform)

<table>
<thead>
<tr>
<th></th>
<th>Earth</th>
<th>Saturn</th>
</tr>
</thead>
<tbody>
<tr>
<td>IFT using Earth-magnitude &amp; Saturn-phase</td>
<td>IFT using Earth-magnitude &amp; nil-phase</td>
<td>IFT using Saturn-magnitude &amp; Earth-phase</td>
</tr>
</tbody>
</table>
Theodor Kaluza (1921) developed a math extension of the classical general relativity theory to 5D. This included the metric, the field equations, the equations of motion, the stress-energy tensor, and the cylinder condition. Physicist Oskar Klein (1926) interpreted Kaluza’s 3D+2D theory in quantum mechanical space and proposed that the fifth dimension was curled up and microscopic.

The topology of the 5D Kaluza-Klein spacetime is $K_2 \cong M^4 \times S^1$, where $M^4$ is a 4D Minkowski spacetime and $S^1$ is a circle (non-traversable).

At a given spatial location, $x$, complex time ($kime$) is defined by $\kappa = r e^{i \phi} \in \mathbb{C}$, where:

- the magnitude represents the longitudinal events order ($r > 0$) and characterizes the longitudinal displacement in time, and
- event phase ($-\pi \leq \phi < \pi$) is an angular displacement, or event direction

There are multiple alternative parametrizations of kime in the complex plane

Space-kime manifold is $\mathbb{R}^3 \times \mathbb{C}$:

- $(x, k_1)$ and $(x, k_4)$ have the same spacetime representation, but different spacekime coordinates,
- $(x, k_1)$ and $(y, k_1)$ share the same kime, but represent different spatial locations,
- $(x, k_2)$ and $(x, k_3)$ have the same spatial-locations and kime-directions, but appear sequentially in order, $r_2 < r_1$. 
Kime Parameterizations

The Spacekime Manifold

- **Spacekime:** \((x, k) = \left( x^1, x^2, x^3, c_k x^4, c_k x^5 \right) \in X, \ c \sim 3 \times 10^8 \text{ m/s} \)

- **Ke vents** (complex events): points (or states) in the spacekime manifold \(X\). Each kevent is defined by where \((x = (x, y, z))\) it occurs in space, what is its causal longitudinal order \(r = x^4 + x^5\), and in what kime-direction \((\varphi = \arctan(2z/y))\) it takes place.

- **Spacekime interval** \((ds)\) is defined using the general Minkowski \(5 \times 5\) metric tensor \((\lambda_{ij})_{i=1,j=1}^{5,5}\), which characterizes the geometry of the (generally curved) spacekime manifold:
  \[
  ds^2 = \sum_{i=1}^{5} \sum_{j=1}^{5} \lambda_{ij} dx^i dx^j = \lambda_{ij} dx^i dx^j
  \]

- **Euclidean (flat) spacekime** metric corresponds to the tensor:
  \[
  \lambda_{ij} = \begin{bmatrix}
  1 & 0 & 0 & 0 & 0 \\
  0 & 1 & 0 & 0 & 0 \\
  0 & 0 & 1 & 0 & 0 \\
  0 & 0 & 0 & 1 & 0 \\
  0 & 0 & 0 & 0 & -1
  \end{bmatrix}
  \]

- Spacelike intervals correspond to \(ds^2 > 0\), where an inertial frame can be found such that two kevents \(a, b \in X\) are simultaneous. An object can’t be present at two kevents which are separated by a spacelike interval.

- Lightlike intervals correspond to \(ds^2 = 0\). If two kevents are on the line of a photon, then they are separated by a lightlike interval and a ray of light could travel between the two kevents.

- Kimelike intervals correspond to \(ds^2 < 0\). An object can be present at two different kevents, which are separated by a kimelike interval.
Spacekime Calculus

- **Wirtinger derivative** (first order kime-derivative at $k = (r, \varphi)$):
  
  $$f'(z) = \frac{\partial f(z)}{\partial z} = \frac{1}{2} \left( \frac{\partial f}{\partial x} - i \frac{\partial f}{\partial y} \right) \quad \text{and} \quad f'(\bar{z}) = \frac{\partial f(\bar{z})}{\partial \bar{z}} = \frac{1}{2} \left( \frac{\partial f}{\partial x} + i \frac{\partial f}{\partial y} \right).$$

In Conjugate-pair basis:

$$df = \partial f + \partial \bar{f} = \frac{\partial f}{\partial z} dz + \frac{\partial \bar{f}}{\partial \bar{z}} d\bar{z}.$$

In Polar kime coordinates:

$$f'(k) = \frac{1}{2} \left( \cos \varphi \frac{\partial f}{\partial r} - \frac{1}{r} \sin \varphi \frac{\partial f}{\partial \varphi} - i \left( \sin \varphi \frac{\partial f}{\partial r} + \frac{1}{r} \cos \varphi \frac{\partial f}{\partial \varphi} \right) \right) e^{-i \varphi} \left( \frac{\partial f}{\partial r} - \frac{1}{r} \frac{\partial f}{\partial \varphi} \right).$$

$$f'(\bar{k}) = \frac{1}{2} \left( \cos \varphi \frac{\partial \bar{f}}{\partial r} - \frac{1}{r} \sin \varphi \frac{\partial \bar{f}}{\partial \varphi} + i \left( \sin \varphi \frac{\partial \bar{f}}{\partial r} + \frac{1}{r} \cos \varphi \frac{\partial \bar{f}}{\partial \varphi} \right) \right) e^{i \varphi} \left( \frac{\partial \bar{f}}{\partial r} + \frac{1}{r} \frac{\partial \bar{f}}{\partial \varphi} \right).$$

- **Wirtinger acceleration** (second order kime-derivative at $k = (r, \varphi)$):

$$f''(k) = \frac{1}{4r^2} \left( \cos \varphi - i \sin \varphi \right)^2 \left( 2 \frac{\partial f}{\partial \varphi} \frac{\partial^2 f}{\partial \varphi^2} + r \left( - \frac{\partial f}{\partial r} - 2 \frac{\partial^2 f}{\partial r \partial \varphi} + r \frac{\partial^2 f}{\partial r^2} \right) \right).$$

**Dinov & Velev (2021)**

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**Spacekime Calculus**

- **Path integral** of a complex function $f: \mathbb{C} \to \mathbb{C}$ on a specific path connecting $z_a \in \mathbb{C}$ to $z_b \in \mathbb{C}$ is defined by generalizing Riemann sums:

$$\lim_{|z_{i+1} - z_i| \to 0} \sum_{i=1}^{n-1} (f(z_i)(z_{i+1} - z_i)) \equiv \int_{z_a}^{z_b} f(z)dz.$$

This assumes the path is a polygonal arc joining $z_a$ to $z_b$, via $z_1 = z_a, z_2, z_3, \ldots, z_n = z_b$, and we integrate the piecewise constant function $f(z_i)$ on the arc joining $z_i \rightarrow z_{i+1}$.

**Assumptions**: the path $z_a \rightarrow z_b$ needs to be defined and the limit of the generalized Riemann sums, as $n \to \infty$, will yield a complex number representing the Wirtinger integral of the function over the path.

- Similarly, extend the classical area integrals, indefinite integral, and Laplacian:

  **Definite area integral**: for $\Omega \subseteq \mathbb{C}$, $\int_{\Omega} f(z)dz$.  

  **Indefinite integral**: $\int f(z)dz = \frac{\partial f}{\partial z}dz + \frac{\partial f}{\partial \bar{z}}d\bar{z}$. 

  The **Laplacian** in terms of conjugate pair coordinates is $\Delta f = \nabla^2 f = 4 \frac{\partial f}{\partial z} \frac{\partial f}{\partial z} dz = 4 \frac{\partial f}{\partial z} \frac{\partial f}{\partial \bar{z}} d\bar{z} d\bar{z}$.

**Dinov & Velev (2021)**
Newton's equations of motion in kime

\[ \begin{align*}
  v &= at + v_0, \\
  x &= x_0 + v_0 t + \frac{1}{2} \alpha t^2 \\
  v^2 &= 2\alpha (x - x_0) + v_0^2 \\
\end{align*} \]

\[ \begin{align*}
  v &= a_1 k_1 + v_{01} = a_2 k_2 + v_{02}, \\
  x &= x_{01} + v_{01} k_1 + \frac{1}{2} a_1 k_1^2 = x_{02} + v_{02} k_2 + \frac{1}{2} a_2 k_2^2, \\
  \sqrt{v_{02}^2 - v_{01}^2} &= a_1 (x - x_{01}) + \sqrt{v_{02}^2 - v_{01}^2}, \\
  \sqrt{v_{01}^2 - v_{02}^2} &= a_2 (x - x_{02}) + \sqrt{v_{01}^2 - v_{02}^2}. \\
\end{align*} \]

Derived from the Kime Wirtinger velocity and acceleration

Kime-velocity \((k = (t, \varphi))\) is defined by the Wirtinger derivative of the position with respect to kime:

\[ v(k) = \frac{\partial x}{\partial k} = \frac{1}{2} \left( \cos \varphi \frac{\partial x}{\partial t} - \frac{1}{1 - \beta^2} \sin \varphi \frac{\partial x}{\partial \varphi} \right) \]

The directional kime derivatives \(v_1\) and \(v_2\).

Spacekime Generalizations

Spacekime generalization of Lorentz transform between two reference frames, \(K'\) & \(K''\):

\[ \begin{pmatrix}
  x' \\
  y' \\
  z' \\
  k_{1}' \\
  k_{2}'
\end{pmatrix}
= \begin{pmatrix}
  \zeta & 0 & 0 & -\frac{c^2}{v_1} \beta^2 \zeta & -\frac{c^2}{v_2} \beta^2 \zeta \\
  0 & 1 & 0 & \frac{c^2}{v_1} \beta^2 \zeta & 0 \\
  0 & 0 & 1 & 0 & \frac{c^2}{v_2} \beta^2 \zeta \\
  0 & 0 & 0 & 1 + (\zeta - 1) \frac{c^2}{v_1 v_2} \beta^2 & (\zeta - 1) \frac{c^2}{v_1 v_2} \beta^2 \\
  0 & 0 & 0 & (\zeta - 1) \frac{c^2}{v_1 v_2} \beta^2 & 1 + (\zeta - 1) \frac{c^2}{v_2^2} \beta^2
\end{pmatrix}
\begin{pmatrix}
  x \\
  y \\
  z \\
  k_{1} \\
  k_{2}
\end{pmatrix} \]

where \(0 \leq \beta = \frac{1}{\sqrt{\left(\frac{c}{v_1}\right)^2 + \left(\frac{c}{v_2}\right)^2}} \leq 1\) & \(\zeta = \frac{1}{\sqrt{1 - \beta^2}} \geq 1\).
Spacekime Solution to Wave Equation

- Math Generalizations
- Derived other spacekime concepts: law of addition of velocities, energy-momentum conservation law, stability conditions for particles moving in spacekime, conditions for nonzero rest particle mass, causal structure of spacekime, and solutions of the ultrahyperbolic wave equation under Cauchy initial data ...

Wang et al., 2021 | Dinov & Velev (2021)

Ultrahyperbolic Wave Equation – Cauchy Initial Data

- Nonlocal constraints yield the existence, uniqueness & stability of local and global solutions to the ultrahyperbolic wave equation under Cauchy initial data ...

\[
\sum_{i=1}^{d_s} \partial_{x_i}^2 u \equiv \Delta_s u(x, \kappa) = \Delta_t u(x, \kappa) \equiv \sum_{i=1}^{d_t} \partial_{\zeta_i}^2 u, \\
\begin{align*}
\frac{\partial}{\partial \zeta_1} u(x,0,\kappa,\eta) &= f(x,\kappa_{-1}), \\
\frac{\partial}{\partial \zeta_1} u(x,0,\kappa,\eta) &= g(x,\kappa_{-1}),
\end{align*}
\]

where \( x = (x_1, x_2, ..., x_{d_s}) \in \mathbb{R}^{d_s} \) and \( \kappa = (\kappa_1, \kappa_2, ..., \kappa_{d_t}) \in \mathbb{R}^{d_t} \) are the Cartesian coordinates in the \( d_s \) space and \( d_t \) time dims.

Stable local solution over a Fourier frequency region defined by \( |\xi| \geq |\eta_{-1}| \) — nonlocal constraints:

\[
\tilde{u}(\xi, \eta_{-1}) = \cos \left( 2\pi \kappa_1 \sqrt{|\xi|^2 - |\eta_{-1}|^2} \right) \tilde{u}_0(\xi, \eta_{-1}) + \sin \left( 2\pi \kappa_1 \sqrt{|\xi|^2 - |\eta_{-1}|^2} \right) \tilde{u}_1(\xi, \eta_{-1}),
\]

where

\[
\mathcal{F}\left( \frac{\partial}{\partial \zeta_1} u \right) = \left( \frac{\partial}{\partial \zeta_1} \tilde{u} \right) = \left( \frac{\partial}{\partial \zeta_1} \tilde{u}(\xi, \eta_{-1}) \right) \bigg|_{\xi} - \left( \frac{\partial}{\partial \zeta_1} \tilde{u}(\xi, \eta_{-1}) \right) \bigg|_{\eta_{-1}}.
\]

\[
\frac{\partial}{\partial \zeta_1} u \left( x, \kappa_{-1}, \eta_{-1} \right) = \mathcal{F}^{-1}(\tilde{u})(x, \kappa) = \int_{\mathbb{R}^{d_s} \times \mathbb{R}^{d_t}} \tilde{u}(\xi, \kappa_1, \eta_{-1}) \times e^{2\pi i \langle x, \xi \rangle} \times e^{2\pi i \langle \kappa_{-1}, \eta_{-1} \rangle} d\xi \, d\eta_{-1}.
\]

Wang et al., 2021 | Dinov & Velev (2021)
Hidden Variable Theory & Random Sampling

- Kime phase distributions are mostly symmetric, random observations \(\equiv\) phase sampling

![Sample the 2D Kime Manifold parameterized by \((\mu, \theta)\)](image)

\[\mu_{01} = \frac{\pi}{5}, \quad \mu_{02} = 0, \quad \mu_{03} = -\frac{\pi}{3}\]


Kime-Phase Sampling Simulation

![Trivariate random sampling of kime-magnitudes (times) and kime-directions (phases)](image)

https://Spacekime.org  Dinov & Velev (2021)
(Many) Spacekime Open Math Problems

- **Ergodicity**

Let’s look at the particle velocities in the 4D Minkowski spacetime \((X)\), a measure space where gas particles move spatially and evolve longitudinally in time. Let \(\mu = \mu_x\) be a measure on \(X\), \(f(x, t) \in L^1(X, \mu)\) be an integrable function (e.g., velocity of a particle), and \(T_t: X \to X\) be a measure-preserving transformation at position \(x \in \mathbb{R}^3\) at time \(t \in \mathbb{R}^+\).

Prove a pointwise ergodic theorem arguing that in a measure theoretic sense, the average of \(f\) over all particles in the gas system at a fixed time, \(\bar{f} = \bar{E}_t(f) = \int_{\mathbb{R}^3} f(x, t) d\mu_x\), will be equal to the average velocity (\(\bar{f}\)) of just one particle (\(x\)) over the entire time span,

\[
\bar{f} = \lim_{n\to\infty} \left( \frac{1}{n} \sum_{i=0}^n f(T^i(x)) \right).
\]

That is, prove that \(\bar{f} \equiv \bar{f}\).

The spatial probability measure is denoted by \(\mu_x\) and the transformation \(T^t x\) represents the dynamics (time evolution) of the particle starting with an initial spatial location \(T^0 x = x\).

Investigate the ergodic properties of various transformations in the 5D spacekime:

\[
\bar{f} \equiv E_\kappa(f) = \frac{1}{\mu_x(X)} \int f(x, t, \phi) d\mu_x \equiv \lim_{t\to\infty} \left( \frac{1}{t} \sum_{i=0}^t \left( f(T^i x, t, \phi) d\Phi \right) \right) \equiv \bar{f}
\]

Dinov & Velev (2021)

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Spacekime Connection to Data Science & Neuroscience?
### Mathematical-Physics $\implies$ Data/Neuro Sciences

<table>
<thead>
<tr>
<th>Mathematical-Physics</th>
<th>Data/Neuro Sciences</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A particle</strong> is a small localized object that permits observations and characterization of its physical or chemical properties</td>
<td><strong>An object</strong> is something that exists by itself, actually or potentially, concretely or abstractly, physically or incorporeal (e.g., person, subject, etc.)</td>
</tr>
<tr>
<td>An <strong>observable</strong> a dynamic variable about particles that can be measured</td>
<td>A <strong>feature</strong> is a dynamic variable or an attribute about an object that can be measured</td>
</tr>
<tr>
<td>Particle <strong>state</strong> is an observable particle characteristic (e.g., position, momentum)</td>
<td><strong>Datum</strong> is an observed quantitative or qualitative value, an instantiation, of a feature</td>
</tr>
<tr>
<td>Particle <strong>system</strong> is a collection of independent particles and observable characteristics, in a closed system</td>
<td><strong>Problem</strong>, aka Data System, is a collection of independent objects and features, without necessarily being associated with apriori hypotheses</td>
</tr>
<tr>
<td><strong>Wave-function</strong></td>
<td><strong>Inference-function</strong></td>
</tr>
<tr>
<td>Reference-Frame transforms (e.g., Lorentz)</td>
<td>Data transformations (e.g., wrangling, log-transform)</td>
</tr>
<tr>
<td><strong>State of a system</strong> is an observed measurement of all particles – wavefunction</td>
<td><strong>Dataset (data)</strong> is an observed instance of a set of datum elements about the problem system, $O = {X, Y}$</td>
</tr>
<tr>
<td>A particle <strong>system is computable</strong> if (1) the entire system is logical, consistent, complete and (2) the unknown internal states of the system don’t influence the computation (wavefunction, intervals, probabilities, etc.)</td>
<td><strong>Computable data object</strong> is a very special representation of a dataset which allows direct application of computational processing, modeling, analytics, or inference based on the observed dataset</td>
</tr>
</tbody>
</table>

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### Mathematical-Physics $\implies$ Data Science

<table>
<thead>
<tr>
<th>Math-Physics</th>
<th>Data Science</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Wavefunction</strong></td>
<td><strong>Inference function</strong> - describing a solution to a specific data analytic system (a problem). For example,</td>
</tr>
<tr>
<td><strong>Wave equ problem:</strong></td>
<td><em>A linear (GLM) model</em> represents a solution of a prediction inference problem, $Y = X\beta$, where the inference function quantifies the effects of all independent features ($X$) on the dependent outcome ($Y$), data: $O = {X, Y}$: $\psi(O) = \psi(X, Y) \Rightarrow \beta = \hat{\beta}_{OLS} = (X'X)^{-1}X'Y$.</td>
</tr>
<tr>
<td>$\left(\frac{\partial^2}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right)\psi(x, t) = 0$</td>
<td><em>A non-parametric, non-linear, alternative inference is SVM classification. If $\psi_x \in H$, is the lifting function $\psi: R^d \rightarrow R^d (\psi: x \in R^d \rightarrow \tilde{x} = \psi_x \in H)$, where $d \ll d$, the kernel $\phi(x) = (x(y): O \times O \rightarrow R$ transforms non-linear to linear separation, the observed data $O = {x_i, y_i} \in R^d$ are lifted to $\psi_{O_i} \in H$. Then, the SVM prediction operator is the weighted sum of the kernel functions at $\psi_{O_i}$, where $\beta^</em>$ is a solution to the SVM regularized optimization: $\langle \psi_0</td>
</tr>
</tbody>
</table>
| Complex Solution: $\psi(x, t) = Ae^{i(kx - wt)}$ | The linear coefficients, $p_i$, are the dual weights that are multiplied by the label corresponding to each training instance, $(y_i)$.

where $|w| / k = v$, represents a traveling wave | *Inference always depends on the (input) data; however, it does not have 1-1 and onto bijective correspondence with the data, since the inference function quantifies predictions in a probabilistic sense.* |

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Spacekime Analytics

- Let’s assume that we have:
  1. Kime extension of Time, and
  2. Parallels between wavefunctions ↔ inference functions
- Often, we can’t directly observe (record) data natively in 5D spacekime.
- Yet, we can measure quite accurately the kime-magnitudes ($r$) as event orders, “times”.
- To reconstruct the 2D spatial structure of kime, borrow tricks used by crystallographers to resolve the structure of atomic particles by only observing the magnitudes of the diffraction pattern in k-space. This approach heavily relies on (1) prior information about the kime directional orientation (that may be obtained from using similar datasets and phase-aggregator analytical strategies), or (2) experimental reproducibility by repeated confirmations of the data analytic results using longitudinal datasets.


Spacekime Analytics: fMRI Example

- 3D Isosurface Reconstruction of $(\text{space}=2, \text{time}=1)$ fMRI signal

4D spacetime; Reconstruction using trivial phase-angle; kime=time=$(\text{magnitude}, 0)$

5D Spacekime; Reconstruction using correct kime=$(\text{magnitude}, \text{phase})$

$3D \text{ pseudo-spacetime reconstruction:}$

$$f = \hat{h} \left( \frac{x_1, x_2, t}{\text{space}, \text{time}} \right)$$
**Spacekime Analytics:**

**Kime-series = Surfaces (not curves)**

In the 5D spacekime manifold, time-series curves extend to kime-series, i.e., surfaces parameterized by kime-magnitude ($t$) and the kime-phase ($\varphi$).

Kime-phase aggregating operators that can be used to transform standard time-series curves to spacekime kime-surfaces, which can be modeled, interpreted, and predicted using advanced spacekime analytics.

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**Spacekime Analytics: fMRI kime-series**

fMRI *kime-series* at a single spatial voxel location (represents fMRI kime intensities)

Kime-Foliation:
Specific 1D time-series are projections of kime-series (red & blue curves)
Spacetime Time-series ➔ Spacekime Kime-surfaces

Spacekime Analytics: Demos

- Tutorials
  - https://TCIU.predictive.space
  - https://SpaceKime.org

- R Package
  - https://cran.rstudio.com/web/packages/TCIU

- GitHub
  - https://github.com/SOCR/TCIU
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https://SOCR.umich.edu