Computational Neuroscience, Time Complexity & Spacekime Analytics

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Joint work with Milen V. Velev (BTU)

Based on an upcoming book "Data Science: Time Complexity, Inferential Uncertainty & Spacekime Analytics"

Outline

- Motivation: Big Data Analytics Challenges
- Complex-Time (kime)
- Spacekime Calculus & Math Foundations
- Open Spacekime Problems

- Neuroscience Applications
  - Longitudinal Neuroimaging (UKBB, fMRI)
**Big Data Analytics Challenges**

**Common Characteristics of Big Data**

<table>
<thead>
<tr>
<th>Big Bio Data Dimensions</th>
<th>Tools</th>
<th>Example: analyzing observational data of 1,000's Parkinson's disease patients based on 10,000's signature biomarkers derived from multi-source imaging, genetics, clinical, physiologic, phenomics and demographic data elements</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Size</strong></td>
<td>Harvesting and management of vast amounts of data</td>
<td></td>
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<tr>
<td><strong>Complexity</strong></td>
<td>Wranglers for dealing with heterogeneous data</td>
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<td><strong>Incongruency</strong></td>
<td>Tools for data harmonization and aggregation</td>
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<td><strong>Multi-source</strong></td>
<td>Transfer and joint multivariate representation &amp; modeling</td>
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<td><strong>Multi-scale</strong></td>
<td>Macro → meso → micro → nano scale observations</td>
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<tr>
<td><strong>Time</strong></td>
<td>Techniques accounting for longitudinal effects (e.g., time corr)</td>
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<tr>
<td><strong>Incomplete</strong></td>
<td>Reliable management of missing data, imputation</td>
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</table>

**IBM Big Data 4V's: Volume, Variety, Velocity & Veracity**

**Dinov, GigaScience (2016) PMID:26918190**
Complex-Time (*kime*) & Spacekime Foundations

2D Fourier Transform – The Importance of Magnitudes & Phases

<table>
<thead>
<tr>
<th>Fourier Analysis</th>
<th>Earth</th>
<th>Saturn</th>
</tr>
</thead>
<tbody>
<tr>
<td>2D image 1 (Earth)</td>
<td><img src="image" alt="Magnitude FT(Earth)" /></td>
<td><img src="image" alt="Phase FT(Earth)" /></td>
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<tr>
<td>2D image 2 (Saturn)</td>
<td><img src="image" alt="Magnitude FT(Saturn)" /></td>
<td><img src="image" alt="Phase FT(Saturn)" /></td>
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<th>Fourier Synthesis</th>
<th>Earth</th>
<th>Saturn</th>
</tr>
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<tr>
<td><img src="image" alt="IFT using Earth-magnitude &amp; Saturn-phase" /></td>
<td><img src="image" alt="IFT using Earth-magnitude &amp; nil-phase" /></td>
<td></td>
</tr>
<tr>
<td><img src="image" alt="IFT using Saturn-magnitude &amp; Earth-phase" /></td>
<td><img src="image" alt="IFT using Saturn-magnitude &amp; nil-phase" /></td>
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</tr>
</tbody>
</table>
Kaluza-Klein Theory

- Theodor Kaluza (1921) developed a math extension of the classical general relativity theory to 5D. This included the metric, the field equations, the equations of motion, the stress-energy tensor, and the cylinder condition. Physicist Oskar Klein (1926) interpreted Kaluza’s 3D+2D theory in quantum mechanical space and proposed that the fifth dimension was curled up and microscopic.

- The topology of the 5D Kaluza-Klein spacetime is $K_2 \cong M^4 \times S^1$, where $M^4$ is a 4D Minkowski spacetime and $S^1$ is a circle (non-traversable).

Complex-Time (Kime)

- At a given spatial location, $x$, complex time (kime) is defined by $\kappa = r e^{i\phi} \in \mathbb{C}$, where:
  - the magnitude represents the longitudinal events order ($r > 0$) and characterizes the longitudinal displacement in time, and
  - event phase ($-\pi \leq \phi < \pi$) is an angular displacement, or event direction
- There are multiple alternative parametrizations of kime in the complex plane
- Space-kime manifold is $\mathbb{R}^3 \times \mathbb{C}$:
  - $(x, k_1)$ and $(x, k_4)$ have the same spacetime representation, but different spacekime coordinates,
  - $(x, k_1)$ and $(y, k_4)$ share the same kime, but represent different spatial locations,
  - $(x, k_2)$ and $(x, k_3)$ have the same spatial-locations and kime-directions, but appear sequentially in order, $r_2 < r_1$. 
Math Foundations of Spacekime

- **Spacekime**: \((x, k) = \left(\frac{x^1, x^2, x^3, c\kappa_1 = x^4, c\kappa_2 = x^5}{\text{space}, \text{kime}}\right) \in X\)

- **Kevents (complex events)**: points (or states) in the spacekime manifold \(X\). Each kevent is defined by where \((x = (x, y, z))\) it occurs in space, what is its *causal longitudinal order* \(r = \sqrt{(x^4)^2 + (x^5)^2}\), and in what *kime-direction* \((\phi = \text{atan2}(x^5, x^4))\) it takes place.

- **Spacekime interval** \((ds)\) is defined using the general Minkowski \(5 \times 5\) metric tensor.

- **Spacekime Calculus of differentiation and integration** (defined using Wirtinger derivatives and path integration)

- **Generalization of the equations of motion in spacekime**

- **Lorentz transformation** (between 2 spacekime inertial frames)

- **Solutions to ultrahyperbolic PDEs**

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Spacekime Solution to Wave Equation

- **Math Generalizations**

- **Derived other spacekime concepts**: law of addition of velocities, energy-momentum conservation law, stability conditions for particles moving in spacekime, conditions for nonzero rest particle mass, causal structure of spacekime, and solutions of the ultrahyperbolic wave equation under Cauchy initial data ...
Hidden Variable Theory & Random Sampling

Kime phase distributions are mostly symmetric, random observations ≡ phase sampling


Kime-Phase Sampling Simulation

Dinov & Velev (2021)
Spacekime Connection to Data Science?

Mathematical-Physics $\implies$ Data Science

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<thead>
<tr>
<th>Mathematical-Physics</th>
<th>Data Science</th>
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<tbody>
<tr>
<td>A <strong>particle</strong> is a small localized object that permits observations and characterization of its physical or chemical properties</td>
<td>An <strong>object</strong> is something that exists by itself, actually or potentially, concretely or abstractly, physically or incorporeal (e.g., person, subject, etc.)</td>
</tr>
<tr>
<td>An <strong>observable</strong> a dynamic variable about particles that can be measured</td>
<td>A <strong>feature</strong> is a dynamic variable or an attribute about an object that can be measured</td>
</tr>
<tr>
<td>Particle <strong>state</strong> is an observable particle characteristic (e.g., position, momentum)</td>
<td><strong>Datum</strong> is an observed quantitative or qualitative value, an instantiation, of a feature</td>
</tr>
<tr>
<td>Particle <strong>system</strong> is a collection of independent particles and observable characteristics, in a closed system</td>
<td><strong>Problem</strong>, aka Data System, is a collection of independent objects and features, without necessarily being associated with apriori hypotheses</td>
</tr>
<tr>
<td><strong>Wave-function</strong></td>
<td><strong>Inference-function</strong></td>
</tr>
<tr>
<td>Reference-Frame <strong>transforms</strong> (e.g., Lorentz)</td>
<td>Data <strong>transformations</strong> (e.g., wrangling, log-transform)</td>
</tr>
<tr>
<td><strong>State of a system</strong> is an observed measurement of all particles – wavefunction</td>
<td><strong>Dataset</strong> (data) is an observed instance of a set of datum elements about the problem system, $\mathbf{\Omega} = {\mathbf{X}, \mathbf{Y}}$</td>
</tr>
<tr>
<td>A <strong>particle system is computable</strong> if (1) the entire system is logical, consistent, complete and (2) the unknown internal states of the system don’t influence the computation (wavefunction, intervals, probabilities, etc.)</td>
<td><strong>Computable data object</strong> is a very special representation of a dataset which allows direct application of computational processing, modeling, analytics, or inference based on the observed dataset</td>
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<td>...</td>
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Let’s assume that we have:
(1) Kime extension of Time, and
(2) Parallels between wavefunctions ↔ inference functions

Often, we can’t directly observe (record) data natively in 5D spacekime.
Yet, we can measure quite accurately the kime-magnitudes \( r \) as event orders, “times”.
To reconstruct the 2D spatial structure of kime, borrow tricks used by crystallographers \(^1\) to resolve the structure of atomic particles by only observing the magnitudes of the diffraction pattern in k-space. This approach heavily relies on (1) prior information about the kime directional orientation (that may be obtained from using similar datasets and phase-aggregator analytical strategies), or (2) experimental reproducibility by repeated confirmations of the data analytic results using longitudinal datasets.

\(^1\) Rodriguez, Ivanova, Nature 2015

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### Spacekime Analytics: fMRI Example

- 3D isosurface Reconstruction of (space=2, time=1) fMRI signal

\[
\begin{align*}
\text{3D pseudo-spacetime reconstruction:} \\
\mathbf{f} &= \hat{\mathbf{h}} \left( \mathbf{x}_1, \mathbf{x}_2, t \right) \\
&= \hat{\mathbf{h}} \left( \mathbf{x}_1, \mathbf{x}_2, \frac{t}{\text{time}} \right)
\end{align*}
\]
In the 5D spacekime manifold, time-series curves extend to kime-series, i.e., surfaces parameterized by kime-magnitude ($t$) and the kime-phase ($\varphi$).

Kime-phase aggregating operators that can be used to transform standard time-series curves to spacekime kime-surfaces, which can be modeled, interpreted, and predicted using advanced spacekime analytics.

**Spacekime Analytics: fMRI kime-series**

fMRI *kime-series* at a single spatial voxel location (represents fMRI kime intensities)
Spacetime Time-series ➔ Spacekime Kime-surfaces

Difference for ON & OFF Kime-Surface/Kime-Series at a fixed voxel location

Spacekime Analytics: Demos

- Tutorials
  - [https://TCIU.predictive.space](https://TCIU.predictive.space)
  - [https://SpaceKime.org](https://SpaceKime.org)

- R Package
  - [https://cran.rstudio.com/web/packages/TCIU](https://cran.rstudio.com/web/packages/TCIU)

- GitHub
  - [https://github.com/SOCR/TCIU](https://github.com/SOCR/TCIU)
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Collaborators

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