# Spacekime Analytics

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Joint work with Milen V. Velev (BTU)

Based on an upcoming book "*Data Science: Time Complexity, Inferential Uncertainty & Spacekime Analytics*"

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Slides Online: "SOCR News"

## Outline

- Motivation: Big Data Analytics Challenges
- Complex-Time (*kime*)
- Spacekime Calculus & Math Foundations
- Open Spacekime Problems
- Time Complexity & Inferential Uncertainty (TCIU)
  - R package
- Demo Applications – Longitudinal Spacekime Analytics
  - Neuroimaging (UKBB, fMRI)
  - Air quality (UCI ML Air Quality Dataset )

## Big Data Analytics Challenges

## Data Analytics ≡ Information Encoding/Decoding

- From 23 … to … $2^{23}$ (10M) $\left(\underbrace{23}_{2\,\#'s} \rightarrow \underbrace{2^{23}}_{8\,\#'s}\right)$
- Two centuries of Data Science: 1798 → 2020
- In the 18th century, Henry Cavendish used just 23 observations to answer a fundamental question – "What is the Mass of the Earth?" He estimated very accurately the mean density of the Earth/$H_2O$ (5.483±0.1904 g/cm$^3$)
- In the 21st century to achieve the same scientific impact, matching the reliability and the precision of the Cavendish's 18th century prediction, requires a monumental community effort using massive and complex information often exceeding 10M ($2^{23}$) bytes

Dinov (2016) J MedicalStat

## Common Characteristics of Big Data

IBM Big Data 4V's: Volume, Variety, Velocity & Veracity

| Big Bio Data Dimensions | Tools |
|---|---|
| **Size** | Harvesting and management of vast amounts of data |
| **Complexity** | Wranglers for dealing with heterogeneous data |
| **Incongruency** | Tools for data harmonization and aggregation |
| **Multi-source** | Transfer and joint multivariate representation & modeling |
| **Multi-scale** | Macro → meso → micro → nano scale observations |
| **Time** | Techniques accounting for longitudinal effects (e.g., time corr) |
| **Incomplete** | Reliable management of missing data, imputation |

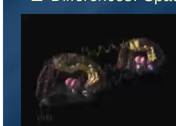
**Example**: analyzing observational data of 1,000's Parkinson's disease patients based on 10,000's signature biomarkers derived from multi-source imaging, genetics, clinical, physiologic, phenomics and demographic data elements

Software developments, student training, service platforms and methodological advances associated with the Big Data Discovery Science all present existing opportunities for learners, educators, researchers, practitioners and policy makers

Dinov, *GigaScience* (2016) PMID:26918190

## Longitudinal Data Analytics

- **Neuroimaging**:
  - *4D fMRI:* time-series, represents measurements of hydrogen atom densities over a 3D lattice of spatial locations ($1 \leq x, y, z \leq 64$ pixels), about 3×3 millimeters² resolution. Data is recorded longitudinally over time ($1 \leq t \leq 180$) in increments of about 3 seconds, then post-processed
  - *State-of-the-art Approaches*: Time-series modeling or Network analysis
  - *Spacekime Analytics*: 5D fMRI kime-series, represent the hydrogen atom densities over the same 3D lattice of spatial locations, longitudinally over the 2D space complex-time (*kime*), $\kappa = re^{i\varphi} \in \mathbb{C}$ (*kimesurfaces*)
  - *Differences*: Spacekime analytics estimate and utilize the kime-phases



4D Spacetime

4D/5D Reconstructions

5D Spacekime

Dinov & Velev (2021)

# Complex-Time (*kime*) & Spacekime Foundations

## The Fourier Transform

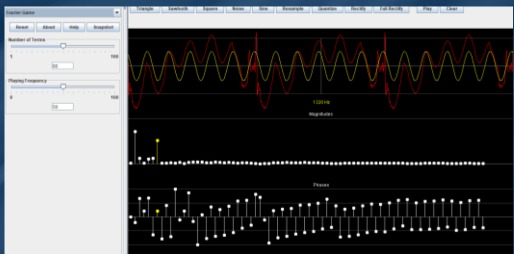
By separability, the classical **spacetime Fourier transform** is just four Fourier transforms, one for each of the four spacetime dimensions, $(\boldsymbol{x}, t) = (x, y, z, t)$. The FT is a function of the angular frequency $\omega$ that propagates in the wave number direction $\boldsymbol{k}$ (space frequency). Symbolically, the forward and inverse Fourier transforms of a 4D ($n = 4$) spacetime function $f$, are defined by:

$$FT(f) = \hat{f}(\boldsymbol{k}, \omega) = \frac{1}{(2\pi)^{\frac{n}{2}}} \int f(\boldsymbol{x}, t) e^{i(\omega t - \boldsymbol{k}\boldsymbol{x})} dt d^3\boldsymbol{x},$$

$$IFT(\hat{f}) = \hat{\hat{f}}(\boldsymbol{x}, t) = \frac{1}{(2\pi)^{\frac{n}{2}}} \int \hat{f}(\boldsymbol{k}, \omega) e^{-i(\omega t - \boldsymbol{k}\boldsymbol{x})} d\omega d^3\boldsymbol{k}.$$

$$\left[\hat{\hat{f}}(\boldsymbol{x}, t) = IFT(\hat{f}) = IFT(FT(f)) = f(\boldsymbol{x}, t), \quad \forall z \in \mathbb{C}, z = \underbrace{A}_{mag} e^{i \overbrace{\varphi}^{phase}}\right]$$

## 1D Fourier Transform Example

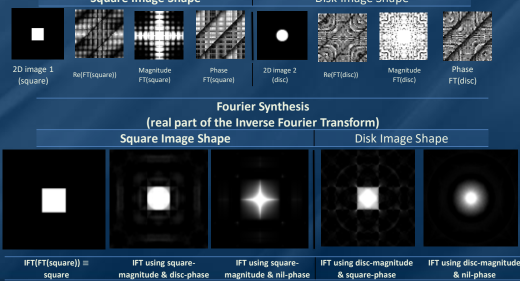


**SOCR 1D Fourier / Wavelet signal decomposition into *magnitudes* and *phases* (*Java applet*)**

Top-panel: original signal (image), white-color curve drawn manually by the user and the reconstructed synthesized (IFT) signal, red-color curve, computed using the user modified magnitudes and phases
Bottom-panels: the Fourier analyzed signal (FT) with its magnitudes and phases

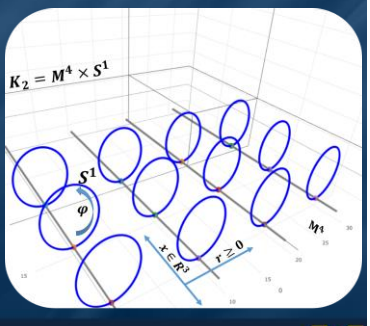
http://www.socr.ucla.edu/htmls/game/Fourier_Game.html (Java Applet)

## 2D Fourier Transform – The Importance of Magnitudes & Phases



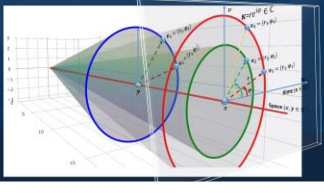

## Kaluza-Klein Theory

- Theodor Kaluza (1921) developed a math extension of the classical general relativity theory to 5D. This included the metric, the field equations, the equations of motion, the stress-energy tensor, and the cylinder condition. Physicist Oskar Klein (1926) interpreted Kaluza's 3D+2D theory in quantum mechanical space and proposed that the fifth dimension was curled up and microscopic.
- The topology of the 5D Kaluza-Klein spacetime is $K_2 \cong M^4 \times S^1$, where $M^4$ is a 4D Minkowski spacetime and $S^1$ is a circle (non-traversable).

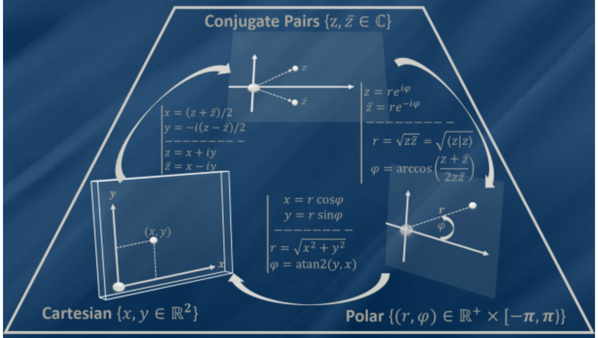



## Complex-Time (*Kime*)

- At a given spatial location, $\boldsymbol{x}$, complex time (*kime*) is defined by $\kappa = re^{i\varphi} \in \mathbb{C}$, where:
  - the magnitude represents the longitudinal events order ($r > 0$) and characterizes the longitudinal displacement in time, and
  - event phase ($-\pi \leq \varphi < \pi$) is an angular displacement, or event direction
- There are multiple alternative parametrizations of kime in the complex plane
- Space-kime manifold is $\mathbb{R}^3 \times \mathbb{C}$:
  - $(\boldsymbol{x}, k_1)$ and $(\boldsymbol{x}, k_4)$ have the same spacetime representation, but different spacekime coordinates,
  - $(\boldsymbol{x}, k_1)$ and $(\boldsymbol{y}, k_1)$ share the same kime, but represent different spatial locations,
  - $(\boldsymbol{x}, k_2)$ and $(\boldsymbol{x}, k_3)$ have the same spatial-locations and kime-directions, but appear sequentially in order, $r_2 < r_1$.



## *Kime Parameterizations*




## The Spacekime Manifold

- **Spacekime:** $(\boldsymbol{x}, \boldsymbol{k}) = \Big(\underbrace{x^1, x^2, x^3}_{\text{space}}, \underbrace{c\kappa_1 = x^4, c\kappa_2 = x^5}_{\text{kime}}\Big) \in X, \quad c \sim 3 \times 10^8\, m/s$
- **Kevents** (*complex events*): points (or states) in the spacekime manifold $X$. Each kevent is defined by where ($\boldsymbol{x} = (x, y, z)$) it occurs in space, what is its *causal longitudinal order* $\left(r = \sqrt{(x^4)^2 + (x^5)^2}\right)$, and in what *kime-direction* ($\varphi = \operatorname{atan2}(x^5, x^4)$) it takes place.
- **Spacekime interval** ($ds$) is defined using the general Minkowski $5 \times 5$ metric tensor $(\lambda_{ij})_{i=1,j=1}^{5,5}$, which characterizes the geometry of the *(generally curved) spacekime* manifold:

$$ds^2 = \sum_{i=1}^{5}\sum_{j=1}^{5} \lambda_{ij}\, dx^i dx^j = \lambda_{ij}\, dx^i dx^j$$

- *Euclidean (flat) spacekime* metric corresponds to the tensor: $(\lambda_{ij}) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix}$
  - Spacelike intervals correspond to $ds^2 > 0$, where an inertial frame can be found such that two kevents $a, b \in X$ are simultaneous. An object can't be present at two kevents which are separated by a spacelike interval.
  - Lightlike intervals correspond to $ds^2 = 0$. If two kevents are on the line of a photon, then they are separated by a lightlike interval and a ray of light could travel between the two kevents.
  - Kimelike intervals correspond to $ds^2 < 0$. An object can be present at two different kevents, which are separated by a kimelike interval.

## Spacekime Calculus

- Kime **Wirtinger derivative** (first order kime-derivative at $\boldsymbol{k} = (r, \varphi)$):

$$f'(z) = \frac{\partial f(z)}{\partial z} = \frac{1}{2}\left(\frac{\partial f}{\partial x} - i\frac{\partial f}{\partial y}\right) \quad \text{and} \quad f'(\bar{z}) = \frac{\partial f(\bar{z})}{\partial \bar{z}} = \frac{1}{2}\left(\frac{\partial f}{\partial x} + i\frac{\partial f}{\partial y}\right).$$

In Conjugate-pair basis: $df = \partial f + \bar{\partial} f = \frac{\partial f}{\partial z}dz + \frac{\partial}{\partial \bar{z}}d\bar{z}$

In Polar kime coordinates:

$$f'(k) = \frac{\partial f(k)}{\partial k} = \frac{1}{2}\left(\cos\varphi\frac{\partial f}{\partial r} - \frac{1}{r}\sin\varphi\frac{\partial f}{\partial \varphi} - i\left(\sin\varphi\frac{\partial f}{\partial r} + \frac{1}{r}\cos\varphi\frac{\partial f}{\partial \varphi}\right)\right) = \frac{e^{-i\varphi}}{2}\left(\frac{\partial f}{\partial r} - \frac{i}{r}\frac{\partial f}{\partial \varphi}\right)$$

$$f'(\bar{k}) = \frac{\partial f(\bar{k})}{\partial \bar{k}} = \frac{1}{2}\left(\cos\varphi\frac{\partial f}{\partial r} - \frac{1}{r}\sin\varphi\frac{\partial f}{\partial \varphi} + i\left(\sin\varphi\frac{\partial f}{\partial r} + \frac{1}{r}\cos\varphi\frac{\partial f}{\partial \varphi}\right)\right) = \frac{e^{i\varphi}}{2}\left(\frac{\partial f}{\partial r} + \frac{i}{r}\frac{\partial f}{\partial \varphi}\right)$$

- Kime **Wirtinger acceleration** (second order kime-derivative at $\boldsymbol{k} = (r, \varphi)$):

$$f''(\boldsymbol{k}) = \frac{1}{4r^2}\left((\cos\varphi - i\sin\varphi)^2\left(2i\frac{\partial f}{\partial \varphi} - \frac{\partial^2 f}{\partial \varphi^2} + r\left(-\frac{\partial f}{\partial r} - 2i\frac{\partial^2 f}{\partial r\partial \varphi} + r\frac{\partial^2 f}{\partial r^2}\right)\right)\right).$$

Dinov & Velev (2021)

## Spacekime Calculus

- Kime **Wirtinger integration**:
  The *path-integral* of a complex function $f: \mathbb{C} \to \mathbb{C}$ on a specific path connecting $z_a \in \mathbb{C}$ to $z_b \in \mathbb{C}$ is defined by generalizing Riemann sums:

$$\lim_{|z_{i+1} - z_i| \to 0} \sum_{i=1}^{n-1} \left(f(z_i)(z_{i+1} - z_i)\right) \cong \oint_{z_a}^{z_b} f(z_i)dz\,.$$

  This assumes the path is a polygonal arc joining $z_a$ to $z_b$, via $z_1 = z_a, z_2, z_3, \dots, z_n = z_b$, and we integrate the piecewise constant function $f(z_i)$ on the arc joining $z_i \to z_{i+1}$. *Assumptions*: the path $z_a \to z_b$ needs to be defined and the limit of the generalized Riemann sums, as $n \to \infty$, will yield a complex number representing the Wirtinger integral of the function over the path.

- Similarly, extend the classical area integrals, indefinite integral, and Laplacian:
  *Definite area integral*: for $\Omega \subseteq \mathbb{C}$, $\int_\Omega f(z)dzd\bar{z}$ .
  *Indefinite integral*: $\int f(z)dzd\bar{z},\ df = \frac{\partial f}{\partial z}dz + \frac{\partial f}{\partial \bar{z}}d\bar{z}$ .
  The *Laplacian* in terms of conjugate pair coordinates is $\Delta f = \nabla^2 f = 4\frac{\partial f}{dz}\frac{\partial f}{d\bar{z}} = 4\frac{\partial f}{d\bar{z}}\frac{\partial f}{dz}$.

Dinov & Velev (2021)

## Newton's equations of motion in kime

$$\left|\begin{aligned} \boldsymbol{v} &= \boldsymbol{a}\boldsymbol{t} + \boldsymbol{v}_o \\ \boldsymbol{x} &= \boldsymbol{x}_o + \boldsymbol{v}_o\boldsymbol{t} + \frac{1}{2}\boldsymbol{a}\boldsymbol{t}^2 \\ \boldsymbol{v}^2 &= 2\boldsymbol{a}(\boldsymbol{x} - \boldsymbol{x}_o) + \boldsymbol{v}_o^2 \end{aligned}\right. \quad \Rightarrow \quad \left|\begin{aligned} &\boldsymbol{v} = \boldsymbol{a}_1\boldsymbol{k}_1 + \boldsymbol{v}_{o1} = \boldsymbol{a}_2\boldsymbol{k}_2 + \boldsymbol{v}_{o2}\,, \\ &\boldsymbol{x} = \boldsymbol{x}_{o1} + \boldsymbol{v}_{o1}\boldsymbol{k}_1 + \frac{1}{2}\boldsymbol{a}_1\boldsymbol{k}_1^2 = \boldsymbol{x}_{o2} + \boldsymbol{v}_{o2}\boldsymbol{k}_2 + \frac{1}{2}\boldsymbol{a}_2\boldsymbol{k}_2^2, \\ &\sqrt{\boldsymbol{v}_2^4 - \boldsymbol{v}^2\boldsymbol{v}_2^2} = -\boldsymbol{a}_1(\boldsymbol{x} - \boldsymbol{x}_{o1}) + \sqrt{\boldsymbol{v}_{o2}^4 - \boldsymbol{v}_o^2\boldsymbol{v}_{o2}^2}, \\ &\sqrt{\boldsymbol{v}_1^4 - \boldsymbol{v}^2\boldsymbol{v}_1^2} = -\boldsymbol{a}_2(\boldsymbol{x} - \boldsymbol{x}_{o2}) + \sqrt{\boldsymbol{v}_{o1}^4 - \boldsymbol{v}_o^2\boldsymbol{v}_{o1}^2} \end{aligned}\right.$$

- Derived from the Kime Wirtinger velocity and acceleration
  - Kime-velocity ($\boldsymbol{k} = (t, \varphi)$) is defined by the Wirtinger derivative of the position with respect to kime:

$$v(\boldsymbol{k}) = \frac{\partial \boldsymbol{x}}{\partial \boldsymbol{k}} = \frac{1}{2}\left(\cos\varphi\frac{\partial \boldsymbol{x}}{\partial t} - \frac{1}{t}\sin\varphi\frac{\partial \boldsymbol{x}}{\partial \varphi} - i\left(\sin\varphi\frac{\partial \boldsymbol{x}}{\partial t} + \frac{1}{t}\cos\varphi\frac{\partial \boldsymbol{x}}{\partial \varphi}\right)\right)$$

  - The directional kime derivatives $v_1$ and $v_2$, (**e** = unit vector of spatial directional change):

$$\boldsymbol{v}_1 = \frac{\sqrt{dx^2 + dy^2 + dz^2}}{dk_1}\boldsymbol{e} = \frac{\sqrt{dx^2 + dy^2 + dz^2}}{\cos\varphi\, dt - t\sin\varphi\, d\varphi}\boldsymbol{e}, \quad \boldsymbol{v}_2 = \frac{\sqrt{dx^2 + dy^2 + dz^2}}{dk_2}\boldsymbol{e} = \frac{\sqrt{dx^2 + dy^2 + dz^2}}{\sin\varphi\, dt + t\cos\varphi\, d\varphi}\boldsymbol{e}$$

Dinov & Velev (2021)

## *Spacekime* Generalizations

- Spacekime generalization of **Lorentz transform** between two reference frames, $K$ & $K'$:

(the interval $ds$ is Lorentz transform invariant)

$$\underbrace{\begin{pmatrix} x' \\ y' \\ z' \\ k_1' \\ k_2' \end{pmatrix}}_{\in K'} = \begin{pmatrix} \zeta & 0 & 0 & -\frac{c^2}{v_1}\beta^2\zeta & -\frac{c^2}{v_2}\beta^2\zeta \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ -\frac{1}{v_1}\beta^2\zeta & 0 & 0 & 1 + (\zeta - 1)\frac{c^2}{(v_1)^2}\beta^2 & (\zeta - 1)\frac{c^2}{v_1 v_2}\beta^2 \\ -\frac{1}{v_2}\beta^2\zeta & 0 & 0 & (\zeta - 1)\frac{c^2}{v_1 v_2}\beta^2 & 1 + (\zeta - 1)\frac{c^2}{(v_2)^2}\beta^2 \end{pmatrix} \underbrace{\begin{pmatrix} x \\ y \\ z \\ k_1 \\ k_2 \end{pmatrix}}_{\in K}$$

where $0 \le \beta = \frac{1}{\sqrt{\left(\frac{c}{v_1}\right)^2 + \left(\frac{c}{v_2}\right)^2}} \le 1$ & $\zeta = \frac{1}{\sqrt{1 - \beta^2}} \ge 1$ .

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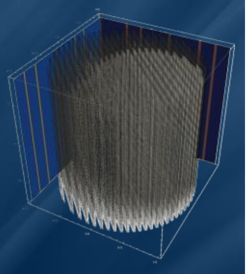
# *Spacekime* Math Generalizations

- Derived other spacekime concepts: law of addition of velocities, energy-momentum conservation law, stability conditions for particles moving in spacekime, conditions for nonzero rest particle mass, causal structure of spacekime, and solutions of the ultrahyperbolic wave equation under Cauchy initial data …



Wang et al., 2021 | Dinov & Velev (2021)

# Ultrahyperbolic Wave Equation – Cauchy Initial Data

- **Nonlocal constraints** yield the existence, uniqueness & stability of local and global solutions to the ultrahyperbolic wave equation under Cauchy initial data …

$$\underbrace{\sum_{i=1}^{d_s} \partial_{x_i}^2 u \equiv \Delta_{\boldsymbol{x}} u(\boldsymbol{x},\boldsymbol{\kappa})}_{\text{spatial Laplacian}} = \underbrace{\Delta_{\boldsymbol{\kappa}} u(\boldsymbol{x},\boldsymbol{\kappa}) \equiv \sum_{i=1}^{d_t} \partial_{\kappa_i}^2 u}_{\text{temporal Laplacian}}, \quad \underbrace{\left| \begin{array}{l} u_o = u\Big(\underbrace{\boldsymbol{x}}_{x\in D_s}, \underbrace{0,\boldsymbol{\kappa}_{-1}}_{\boldsymbol{\kappa}\in D_t}\Big) = f(\boldsymbol{x},\boldsymbol{\kappa}_{-1}) \\ u_1 = \partial_{\kappa_1} u(\boldsymbol{x},0,\boldsymbol{\kappa}_{-1}) = g(\boldsymbol{x},\boldsymbol{\kappa}_{-1}) \end{array}\right.}_{\text{initial conditions (Cauchy Data)}}$$

where $\boldsymbol{x} = (x_1, x_2, \ldots, x_{d_s}) \in \mathbb{R}^{d_s}$ and $\boldsymbol{\kappa} = (\kappa_1, \kappa_2, \ldots, \kappa_{d_t}) \in \mathbb{R}^{d_t}$ are the Cartesian coordinates in the $d_s$ space and $d_t$ time dims.

Stable local solution over a Fourier frequency region defined by $|\boldsymbol{\xi}| \geq |\boldsymbol{\eta}_{-1}|$ … nonlocal constraints:

$$\hat{u}\Big(\boldsymbol{\xi}, \underbrace{\kappa_1, \boldsymbol{\eta}_{-1}}_{\boldsymbol{\eta}}\Big) = \cos\left(2\pi\,\kappa_1 \sqrt{|\boldsymbol{\xi}|^2 - |\boldsymbol{\eta}_{-1}|^2}\right) \underbrace{\hat{u}_o(\boldsymbol{\xi}, \boldsymbol{\eta}_{-1})}_{c_1} + \sin\left(2\pi\,\kappa_1 \sqrt{|\boldsymbol{\xi}|^2 - |\boldsymbol{\eta}_{-1}|^2}\right) \frac{\hat{u}_1(\boldsymbol{\xi}, \boldsymbol{\eta}_{-1})}{\underbrace{2\pi\sqrt{|\boldsymbol{\xi}|^2 - |\boldsymbol{\eta}_{-1}|^2}}_{c_2}},$$

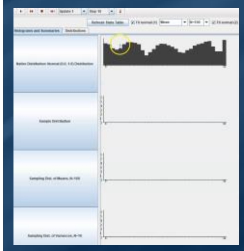
where $\mathcal{F}\begin{pmatrix} u_o \\ u_1 \end{pmatrix} = \begin{pmatrix} \hat{u}_o \\ \hat{u}_1 \end{pmatrix} = \begin{pmatrix} \hat{u}_o(\boldsymbol{\xi}, \boldsymbol{\eta}_{-1}) \\ \hat{u}_1(\boldsymbol{\xi}, \boldsymbol{\eta}_{-1}) \end{pmatrix} = \begin{pmatrix} \hat{u}(\boldsymbol{\xi}, \boldsymbol{\eta}_{-1}) \\ \partial_{\kappa_1} \hat{u}(\boldsymbol{\xi}, \boldsymbol{\eta}_{-1}) \end{pmatrix}$.

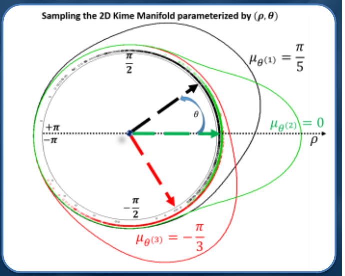
$$u\Big(\underbrace{\boldsymbol{x}, \kappa_1, \boldsymbol{\kappa}_{-1}}_{\boldsymbol{\kappa}}\Big) = \mathcal{F}^{-1}(\hat{u})(\boldsymbol{x},\boldsymbol{\kappa}) = \int_{\hat{D}_s \times \hat{D}_{t_{-1}}} \hat{u}(\boldsymbol{\xi}, \kappa_1, \boldsymbol{\eta}_{-1}) \times e^{2\pi i \langle \boldsymbol{x}, \boldsymbol{\xi}\rangle} \times e^{2\pi i \langle \boldsymbol{\kappa}_{-1}, \boldsymbol{\eta}_{-1}\rangle} d\boldsymbol{\xi}\, d\boldsymbol{\eta}_{-1}.$$

Wang et al., 2021 | Dinov & Velev (2021)

# Hidden Variable Theory & Random Sampling

- Kime phase distributions are mostly symmetric, random observations ≡ phase sampling

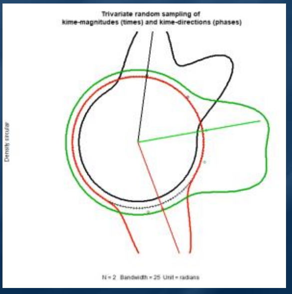





http://wiki.stat.ucla.edu/socr/index.php/SOCR_EduMaterials_Activities_GeneralCentralLimitTheorem

Dinov, Christou & Sanchez (2008) Dinov & Velev (2021)

# Kime-Phase Sampling Simulation




https://Spacekime.org Dinov & Velev (2021)

# *Spacekime* Open Math Problems

- **Ergodicity**

Let's look at the particle velocities in the 4D Minkowski spacetime ($X$), a measure space where gas particles move spatially and evolve longitudinally in time. Let $\mu = \mu_{\boldsymbol{x}}$ be a measure on $X$, $f(\boldsymbol{x}, t) \in L^1(X, \mu)$ be an integrable function (e.g., velocity of a particle), and $T: X \to X$ be a measure-preserving transformation at position $\boldsymbol{x} \in \mathbb{R}^3$ at time $t \in \mathbb{R}^+$.

Prove a pointwise ergodic theorem arguing that in a measure theoretic sense, the average of $f$ over all particles in the gas system at a fixed time, $\bar{f} = E_t(f) = \int_{\mathbb{R}^3} f(\boldsymbol{x}, t) d\mu_{\boldsymbol{x}}$, will be equal to the average velocity ($f$) of just one particle ($\boldsymbol{x}$) over the entire time span,

$\hat{f} = \lim_{n\to\infty} \left(\frac{1}{n}\sum_{i=0}^n f(T^i \boldsymbol{x})\right)$. That is, prove that $\bar{f} \equiv \hat{f}$.

The spatial probability measure is denoted by $\mu_{\boldsymbol{x}}$ and the transformation $T^i\boldsymbol{x}$ represents the dynamics (time evolution) of the particle starting with an initial spatial location $T^o\boldsymbol{x} = \boldsymbol{x}$.

Investigate the ergodic properties of various transformations in the 5D **spacekime**:

$$\underbrace{\bar{f} = E_\kappa(f) = \frac{1}{\mu_{\boldsymbol{x}}(X)} \int f\Big(\boldsymbol{x}, \underbrace{t, \phi}_{\kappa}\Big) d\mu_{\boldsymbol{x}}}_{\text{space averaging}} \overset{?}{\equiv} \underbrace{\lim_{t\to\infty}\left(\frac{1}{t}\sum_{i=0}^{t}\left(\int_{-\pi}^{+\pi} f(T^i\boldsymbol{x}, t, \phi)\, d\Phi\right)\right) = \hat{f}}_{\text{kime averaging}}$$

Dinov & Velev (2021)

# Spacekime Connection to Data Analytics?

## Mathematical-Physics ⟹ Data Science

| Mathematical-Physics | Data Science |
|---|---|
| A **particle** is a small localized object that permits observations and characterization of its physical or chemical properties | An **object** is something that exists by itself, actually or potentially, concretely or abstractly, physically or incorporeal (e.g., person, subject, etc.) |
| An **observable** a dynamic variable about particles that can be measured | A **feature** is a dynamic variable or an attribute about an object that can be measured |
| Particle **state** is an observable particle characteristic (e.g., position, momentum) | **Datum** is an observed quantitative or qualitative value, an instantiation, of a feature |
| Particle **system** is a collection of independent particles and observable characteristics, in a closed system | **Problem**, aka Data System, is a collection of independent objects and features, without necessarily being associated with apriori hypotheses |
| **Wave-function** | **Inference-function** |
| Reference-Frame **transforms** (e.g., Lorentz) | Data **transformations** (e.g., wrangling, log-transform) |
| **State of a system** is an observed measurement of all particles ~ wavefunction | **Dataset (data)** is an observed instance of a set of datum elements about the problem system, $\boldsymbol{O} = \{X, Y\}$ |
| A **particle system is computable** if (1) the entire system is logical, consistent, complete and (2) the unknown internal states of the system don't influence the computation (wavefunction, intervals, probabilities, etc.) | **Computable data object** is a very special representation of a dataset which allows direct application of computational processing, modeling, analytics, or inference based on the observed dataset |
| … | … |

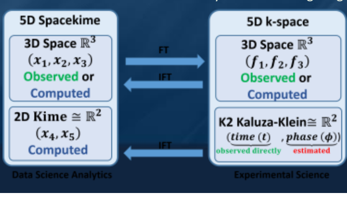
## Mathematical-Physics ⟹ Data Science

| Math-Physics | Data Science |
|---|---|
| *Wavefunction*<br><br>Wave equ problem:<br><br>$\left(\frac{\partial^2}{\partial x^2} - \frac{1}{\nu^2}\frac{\partial^2}{\partial t}\right)\psi(x,t) = 0$<br><br>Complex Solution: $\psi(x,t) = Ae^{i(kx-wt)}$<br>where $\left\lvert\frac{w}{k}\right\rvert = \nu$,<br><br>represents a traveling wave | *Inference function* - describing a solution to a specific data analytic system (a problem). For example,<br>• A linear (GLM) model represents a solution of a prediction inference problem, $Y = X\beta$, where the inference function quantifies the effects of all independent features ($X$) on the dependent outcome ($Y$), data: $\boldsymbol{O} = \{X, Y\}$: $\psi(O) = \psi(X,Y) \Rightarrow \hat{\beta} = \hat{\beta}^{OLS} = \langle X \vert X\rangle^{-1}\langle X\vert Y\rangle = \left(X^T X\right)^{-1}X^T Y$.<br>• A non-parametric, non-linear, alternative inference is SVM classification. If $\psi_x \in H$, is the lifting function $\psi: R^\eta \to R^d$ ($\psi: x \in R^\eta \to \tilde{x} = \psi_x \in H$), where $\eta \ll d$, the kernel $\psi_x(y) = \langle x \vert y\rangle: O \times O \to R$ transformes non-linear to linear separation, the observed data $O_i = \{x_i, y_i\} \in R^\eta$ are lifted to $\psi_{O_i} \in H$. Then, the SVM prediction operator is the weighted sum of the kernel functions at $\psi_{O_i}$, where $\beta^*$ is a solution to the SVM regularized optimization: $\langle \psi_O \vert \beta^*\rangle_H = \sum_{i=1}^{n} p_i^* \langle \psi_O \vert \psi_{O_i}\rangle_H$<br>The linear coefficients, $p_i^*$, are the dual weights that are multiplied by the label corresponding to each training instance, $\{y_i\}$ .<br><br>Inference always depends on the (input) data; however, it does not have 1-1 and onto bijective correspondence with the data, since the inference function quantifies predictions in a probabilistic sense. |

GLM/SVM: https://DSPA.predictive.space | Dinov, Springer (2018)

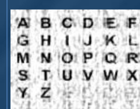
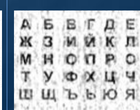
## *Spacekime* Analytics

- Let's assume that we have:
  (1) Kime extension of Time, and
  (2) Parallels between wavefunctions ⟷ inference functions
- Often, we can't directly observe (record) data natively in 5D spacekime.
- Yet, we can measure quite accurately the kime-magnitudes ($r$) as event orders, "times".
- To reconstruct the 2D spatial structure of kime, borrow tricks used by crystallographers [1] to resolve the structure of atomic particles by only observing the magnitudes of the diffraction pattern in k-space. This approach heavily relies on (1) prior information about the kime directional orientation (that may be obtained from using similar datasets and phase-aggregator analytical strategies), or (2) experimental reproducibility by repeated confirmations of the data analytic results using longitudinal datasets.




[1] Rodriguez, Ivanova, Nature 2015

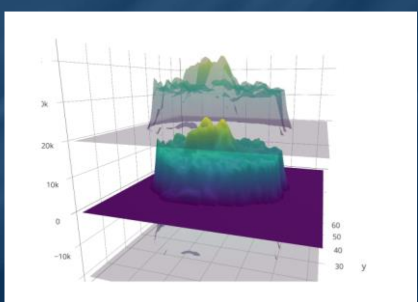
## 2D Image Analysis / Character Recognition

| | | Kime-direction (Phase) Synthesis | | |
|---|---|---|---|---|
| | | Correct Phase | Swapped Phase | Nil-Phase |
| 2D Images | Cyrillic Alphabet |  |  |  |
| | English Alphabet |  |  |  |

Observed Data

Dinov & Velev (2021)

## Back to fMRI (4D spacetime data)



3D rendering of 3 time cross-sections of the fMRI series over a 2D spatial domain

## *Spacekime* Analytics: fMRI Example

- 3D isosurface Reconstruction of (space=2, time=1) fMRI signal

| 4D spacetime: Reconstruction using trivial phase-angle; kime=time=(magnitude, 0) | 5D Spacekime: Reconstruction using correct kime=(magnitude, phase) |
|---|---|

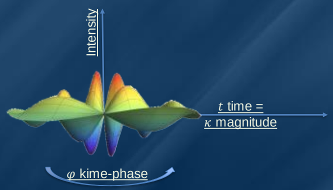
3D pseudo-spacetime reconstruction:

$$f = \hat{h}\,(\underbrace{x_1, x_2}_{space}, \underbrace{t}_{time})$$

## *Spacekime* Analytics: Kime-series = Surfaces (not curves)

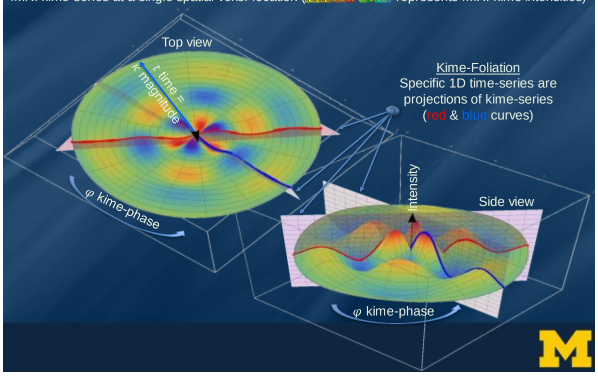
In the 5D spacekime manifold, time-series curves extend to kime-series, i.e., surfaces parameterized by kime-magnitude ($t$) and the kime-phase ($\varphi$).

Kime-phase aggregating operators that can be used to transform standard time-series curves to spacekime kime-surfaces, which can be modeled, interpreted, and predicted using advanced spacekime analytics.




## *Spacekime* Analytics: fMRI kime-series

fMRI *kime-series* at a single spatial voxel location (rainbow color represents fMRI kime intensities)




## *Spacekime* Analytics: **Demos**

- Tutorials
  - https://TCIU.predictive.space
  - https://SpaceKime.org
- R Package
  - https://cran.rstudio.com/web/packages/TCIU
- GitHub
  - https://github.com/SOCR/TCIU

## Interested in Spacekime Analytics?

- Check www.SpaceKime.org
- Contact me
- We have lots of "Open Problems"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Slides Online: "SOCR News"

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https://SOCR.umich.edu